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Cost, Revenue and Profit Efficiency Evaluation in Downstream Petrochemical Industries with Data Envelopment Analysis Approach with Fuzzy Data

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Abstract

One of the applications of data envelopment analysis is the calculation of cost, revenue and profit efficiency, which is used in the financial analysis of organisations. This analysis makes the managers of the organisations make better decisions against the fluctuations caused by the changes in the prices of production inputs in the competitive market, investment risk and other factors effecting their business. In the real world, not all data related to inputs, outputs and their corresponding prices are accurate. Therefore, in order to determine their value, it is necessary to use fuzzy concepts for imprecise data. The purpose of this research is to calculate the cost, revenue and profit efficiency of the production lines of the polymer pipe manufacturing plant from the downstream petrochemical industries with full fuzzy data of the type of triangular fuzzy numbers with an α-cut approach. So that each of the 7 existing production lines is considered as a DMU, this performance evaluation is based on the variety of production lines, product size and limitation in the problem using the data envelopment analysis technique, and then the proposed FDEA model is converted into a family of crisp models to calculate the upper and lower bounds and is ranked based on interval data rules.

Keywords: Data envelopment analysis, Cost, Revenue and profit efficiency, Fuzzy data, α-cut.

1 | Introduction

The data envelopment analysis method is used to measure and analyse some concepts such as cost, revenue and profit efficiency. In fact, one of the most important aspects of analysing the performance of organisations

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is the measurement of cost, revenue and profit efficiency. The cost efficiency model seeks to find a unit that spends the minimum cost to buy inputs that are more than the inputs of the unit under evaluation to produce outputs equal to the outputs of the unit under evaluation. Also, in terms of revenue efficiency, the units are efficient which by consuming inputs equal to the inputs under evaluation, get the most revenue from the sale of outputs less than the outputs of the unit under evaluation.

For the first time, Fare et. al [1] developed a method to implement cost and revenue efficiencies empirically. Charnes et al. [2] generalised the problem of sensitivity analysis to data envelopment analysis models with one output. Efficiency models based on DEA introduced by Fare et al. [1] not only need input and output values, but prices can be different from one unit to another, which can create limitations. In all these models, there are a number of simplifying assumptions.

First, the inputs must be homogeneous, and the given prices must be precisely specified. Changes in the process or characteristics of the inputs cause the inputs of large-sized organisations to be different from the inputs of small-sized organisations. As a result, the inputs and, subsequently, their prices are also different from each other. Secondly, the efficiency measured can have more limited value in real applications even when the inputs are homogeneous because the resulting efficiency measure only reflects the inefficiency of the inputs and does not show the inefficiency of the market (Price). Camanho and Dyson [3] proposed a more comprehensive model that covers both inefficiencies. Thirdly, in reality, the data related to the price in the inputs and outputs are made artificially and represent the average price and don't show the marginal prices (Margin of profit).

However, in the real world, the values of all inputs and outputs are not always accurate, and it is difficult to determine their value. Therefore, the cost, revenue and profit efficiency should be investigated under the uncertainly of the data. To solve the DEA problems in the fuzzy environment, the α-cut method, fuzzy ranking and defuzzification are used. For the first time, Cooper et al. [4] proposed the concept of imprecise data in DEA.

Kao and Liu. [5] used the α-cut method to measure the efficiency of BCC models in the fuzzy environment, and by using different α, they converted the fuzzy DEA model into a family of crisp numbers. Jahanshahloo et al. [6] investigated the application of the cost efficiency model with fuzzy input and output and accurate input prices in insurance organisations and, in another article in [7], extended the concepts of fuzzy DEA and continued the classic cost efficiency model, presented with fuzzy and imprecise prices. Aghayi [8] fully explained the fuzzy cost efficiency model in three different scenarios and solved the model with the α-cut method. In [9], [10], profit efficiency with fuzzy data. Bagherzadeh Valami [11] evaluated the CE model for the case where input and output data are exact and prices are fuzzy.

Ashrafi [12] measured the cost, revenue and profit efficiency model with fuzzy data. Song et al. [9] expressed the profit distribution for reverse logistics systems based on fuzzy efficiency and assuming that the input and output data are imprecise, they built the efficiency measurement model based on FDEA and then proposed a modified shapley value model for the fair distribution of profit. Pourmand and Bafekr Sharak [13] presented the cost efficiency model for its dual form in the fuzzy environment and ranked the obtained results in interval form.

In this article, the cost, revenue and profit efficiency of the production lines of the polymer pipe production plant from the downstream petrochemical industries have been calculated with full fuzzy data of the type of triangular fuzzy number with the α-cut approach. So that each of the 7 existing production lines is considered as a DMU, this performance evaluation is based on the variety of production lines, the size of the products and the limitations in the problem using the technique of data envelopment analysis, and then the proposed FDEA model is converted to a family of crisp models to calculate the upper and lower bounds, and it is ranked based on interval data rules.

2 | Preliminaries

2.1 | Data Envelopment Analysis

Assume there are n Decision-Making Units (DMU), DMU_j, $j = \{1,2, \dots n\}$ with output and input vectors (xj, yj) that $x_j \in \mathbb{R}_+^m$, $y_j \in \mathbb{R}_+^s$ whose production possibility set is defined as follows:

$$PPS = \{(x, y) | \sum_{j=1}^{n} x_j\lambda_j \le x , \sum_{j=1}^{n} y_j\lambda_j \ge y, \lambda \in \Lambda\}.$$

The general DEA model for calculating input efficiency score is as follows:

$$\begin{aligned} &\theta_\rho^* = \min\theta, \\ &s.t, \\ &\sum_{j=1}^{n} \lambda_j x_{ij} \le \theta\, x_{ip,} \qquad i = 1,2, \dots, m, \\ &\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{rp}, \qquad r = 1,2, \dots, s, \\ &\lambda_j \in \Lambda, \qquad j, 1, \dots n. \end{aligned} \tag{1}$$

(λ^*, θ_P^*) As the optimal solution of *Model (1)*. If $\theta_P^* = 1$, Then DMUp is efficient (weak efficiency). Otherwise, DMUp is inefficient.

$$\begin{aligned} &\varphi_P^* = \mathrm{Max}\varphi, \\ &s.t, \\ &\sum_{j=1}^{n} \lambda_j x_{ij} \le x_{ip,} \qquad i = 1,2, \dots, m, \\ &\sum_{j=1}^{n} \lambda_j\, y_{rj} \ge \varphi y_{rp,} \qquad r = 1,2, \dots, s, \\ &\lambda_j \in \Lambda, \qquad j = 1,2, \dots, n. \end{aligned} \tag{2}$$

(λ^*, φ_P^*) As the optimal solution of the *Model (2)*. If $\varphi_P^* = 1$, Then DMUp is efficient (weak efficiency). Otherwise, DMUp is inefficient.

Models (1) and *(2)* are input and output envelope forms, respectively, θ_P^* is called efficiency in input-oriented and $1/\varphi_P^*$ is called efficiency in output-oriented.

2.2 | Cost Efficiency

One of the branches of performance evaluation is cost efficiency review. Cost efficiency evaluates the ability to produce the current amount of output at the lowest amount of cost. In other words, The ratio of minimum cost to current cost is called cost efficiency. Cost efficiency concepts and theory are attributed to Farrell, who is the founder of many concepts in DEA; according to Farrell's definition, there is a need for quantitative input and output data, and it is equally necessary to know the exact price or cost of inputs for each DMU, For this purpose and to calculate the cost efficiency, Farrell proposed the following model which is in constant returns to scale technology.

Assume that $C \in \mathbb{R}^m$ is the price of the input vector. The production cost of DMUp, $p \in \{1,2, \dots, n\}$ is calculated with limited input and output (x_P, y_P) as $c^T x_p = \sum_{i=1}^{m} c_i\, x_{ip}$, So the cost efficiency of DMUp is:

$$\begin{aligned} &\mathrm{Min} \sum_{i=1}^{m} c_i\, x_i\, , \\ &s.t, \end{aligned} \tag{3}$$

$\sum_{j=1}^{n} \lambda_j x_{ij} \leq \quad x_i, \qquad i = 1, \ldots, m,$

$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rp}, \qquad r = 1, \ldots, s,$

$\lambda_j \geq 0, \qquad j = 1, \ldots, n,$

$, x_i \geq 0, \qquad i = 1, \ldots, m.$

(λ^*, x^*) Is the optimal solution of *Model (3)*.

The cost efficiency of DMUp is the ratio of the minimum cost to the actual cost according to the *Model (4)*.

$$CEp = \frac{c^T x^*}{c^T x_p} = \frac{\sum_{i=1}^{m} c_i x_i *}{\sum_{i=1}^{m} c_i x_{ip}}. \tag{4}$$

Definition 1. DMUp is a cost-efficient unit if the value of CEp for it is equal to one

Also, if we write the dual model or multiple modes of *Model (3)*:

$$CE_P = \text{Max} \sum_{r=1}^{s} u_r y_{rp},$$

$$s.t,$$

$$\sum_{r=1}^{s} u_r y_{rJ} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \qquad j = 1, \ldots, n,$$

$$v_i \leq c_i, \qquad i = 1, \ldots, m,$$

$$u_r \geq 0, \qquad r = 1, \ldots, s,$$

$$v_i \geq 0, \qquad i = 1, \ldots, m.$$

2.3 | Revenue Efficiency

Evaluating the revenue efficiency of units is one of the most important evaluations that can provide valuable information about the units. When instead of the input price, we have the output price of the unit under evaluation. In other words, the ratio of maximum revenue to current revenue is called revenue efficiency.

Assume that $P \in \mathbb{R}^s$ is the price of the output vector. The actual revenue from DMUp, $p \in \{1,2, \ldots, n\}$ is $P^T x_p = \sum_{r=1}^{s} P_r y_{rp}$. So, the maximum obtainable output of DMUp is:

$$\text{Max} \sum_{r=1}^{s} p_r y_r,$$

$$s.t, \tag{5}$$

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{ip}, \qquad i = 1, \ldots, m,$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_r, \qquad r = 1, \ldots, s,$$

$$\lambda_j \geq 0, \qquad j = 1, \ldots, n \quad , y_r \geq 0, \qquad r = 1, \ldots, s.$$

(λ^*, y^*) Is the optimal solution of *Model (5)*.

The revenue efficiency of DMUp is the ratio of maximum revenue to the actual revenue according to *Model (6)*.

$$REp = \frac{p^T y^*}{p^T y_p} = \frac{\sum_{r=1}^{s} p_r y_r *}{\sum_{r=1}^{s} p_r y_{rp}}. \tag{6}$$

Definition 2. DMUp is a revenue-efficient unit if the value of REp for it is equal to one

Also, if we write the dual model or multiple modes of *Model (5)*:

$$
\begin{aligned}
&\mathrm{RE_P} = \mathrm{Min} \sum_{i=1}^{m} v_i\, x_{ip},\\
&\mathrm{s.t,}\\
&\sum_{r=1}^{s} u_r\, y_{rJ} - \sum_{i=1}^{m} v_i\, x_{ij} \le 0, \quad j = 1, \dots, n,\\
&u_r \ge p_r, \quad r = 1, \dots, s,\\
&u_r \ge 0, \quad r = 1, \dots, s,\\
&v_i \ge 0, \quad i = 1, \dots, m.
\end{aligned}
\tag{7}
$$

2.4 | Profit Efficiency

$$
\begin{aligned}
&\mathrm{Max} \textstyle\sum_{r=1}^{s} p_r y_r - \sum_{i=1}^{m} c_i x_i,\\
&\mathrm{s.t,}\\
&\textstyle\sum_{j=1}^{n} \lambda_j\, x_{ij} \le x_{i,} \quad i = 1, \dots, m,\\
&\textstyle\sum_{j=1}^{n} \lambda_j\, y_{rj} \ge y_{r,} \quad r = 1, \dots, s,\\
&\lambda_j \ge 0, \quad j = 1, \dots, n,,\\
&y_r \ge 0, \quad x_i \ge 0, \quad r = 1, \dots, s, \quad i = 1, \dots, m
\end{aligned}
\tag{8}
$$

So we have

$$
\mathrm{Pep} = \frac{\sum p_r y_r - \sum c_i x_i}{\sum p_r y_r^* - \sum c_i x_i^*}, \qquad 0 \le \mathrm{Pep} \le 1.
\tag{9}
$$

Definition 3. DMUp is a profit-efficient unit if the value of PEp (8) for it is equal to one. Also, if we write the dual model or multiple modes of *Model (7)*:

$$
\begin{aligned}
&\mathrm{PE_P} = \mathrm{Min} \textstyle\sum_{i=1}^{m} v'_i\, x_{ip} - \sum_{r=1}^{s} u'_r\, y_{rp},\\
&\mathrm{s.t,}\\
&\sum_{r=1}^{s} u_r\, y_{rJ} - \sum_{i=1}^{m} v_i\, x_{ij} \le 0, \quad j = 1, \dots, n,\\
&v_i - v'_i \le c_i, \quad i = 1, \dots, m,\\
&u_r - u'_r \ge p_{r,} \quad r = 1, \dots, s,\\
&v_i \ge 0, \quad i = 1, \dots, m,\\
&u_r \ge 0, \quad r = 1, \dots, s.
\end{aligned}
\tag{10}
$$

2.5 | Triangular Fuzzy Numbers

Fuzzy number $\widetilde{A}$ is a subset of real numbers R with a membership function $\mu_{A^{\sim}}(x)$:

$$\mu_{\tilde{A}}(x): R \to [0,1],$$
$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x) \mid x \in X \ \delta \ \mu_{\tilde{A}}(x): x \to [0,1]\},$$
$$\tilde{A} = (l, m, u),$$
$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \le l, \\ \dfrac{x-l}{m-l}, & l \le x \le m, \\ 1, & x = m, \\ \dfrac{u-x}{u-m}, & m \le x \le u, \\ 0, & x \ge u. \end{cases} \tag{11}$$

Definition 4. α-cut of fuzzy number $\tilde{A}$ is:

$$\tilde{A}a = \{x \in X \mid \mu_{\tilde{A}}(x) \ge \alpha\}, \tag{12}$$

If

$$\tilde{A}a = \{x \in X \mid \mu_{\tilde{A}}(x) > \alpha\}.$$

It is called strong α-cut.

Using the concept of α-cut for triangular fuzzy number, it can be written:

$$\tilde{A}\alpha = [\alpha m + (1-\alpha)l, \alpha m + (1-\alpha)u]. \tag{13}$$

Definition 5. fuzzy number $\tilde{A}$ is called L-R type**.** If its membership function is as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} 1, & \text{for } x \in [a,b], \\ L\left(\dfrac{a-x}{\alpha}\right), & \text{for } x \le a, \\ R\left(\dfrac{x-b}{\beta}\right), & \text{for } x \ge b. \end{cases} \tag{14}$$

Parameters α and β are left and right width, which are non-negative real numbers.

We have from *Definitions 4* and *5*:

$$F.op \to I.op, \qquad \tilde{A}_\alpha = [a,b],$$
$$\text{Max}\tilde{Z} = \sum_{j=1}^{n} \tilde{c}_j \tilde{x}_j,$$
$$\text{s.t},$$
$$\sum_{j=1}^{n} \tilde{a}_{ij}\tilde{x}_j \le \tilde{b}_i, \quad \text{for all } i,$$
$$x \ge 0. \tag{15}$$
$$\text{Max } Z = \sum_{j=1}^{n} [c_j^l, c_j^U]\, x_j,$$
$$\text{s.t},$$
$$\sum_{j=1}^{n} [a_{ij}^l, a_{ij}^U]\, x_j \le [b_i^l, b_i^U], \quad \text{for all } i,$$
$$x \ge 0.$$

$$\begin{aligned} &\text{Max } z^l = \sum_{j=1}^{n} c_j^l x_j, \\ &\sum_{j=1}^{n} a_{ij}^u x_j \leq b_i^l, \quad \text{for all } \ i, \\ &x \geq 0. \end{aligned} \tag{16}$$

$$\begin{aligned} &\text{Max } z^u = \sum_{j=1}^{n} c_j^u x_j, \\ &x_j^u, \\ &\sum_{j=1}^{n} a_{ij}^l x_j \leq b_i^u, \quad \text{for all } i, \\ &x \geq 0. \end{aligned} \tag{17}$$

Theorem 1. In *Model (15)*, suppose that $c_j \in [c_j^l, c_j^U]$, $a_{ij} \in [a_{ij}^l, a_{ij}^u]$ and $b_i \in [b_i^l, b_i^U]$ are arbitrary and z^* is the optimum of the following problem.

$$\begin{aligned} &\text{Max} z^* = \sum_{j=1}^{\prime\prime} c_J x_J, \\ &\text{s.t}, \\ &\sum_{j=1}^{n} a_{ij} x_j \leq b_i, \quad \text{for all } \ i, \\ &x \geq 0, \end{aligned} \tag{18}$$

Then we have

$$z^l \leq \ z^* \leq z^u.$$

Proof: Assume $(z^l, \bar{x})$ is the optimal solution of the *Model (16)*.

$$\sum_{j=1}^{n} a_{ij}\bar{x}_j \leq \sum_{j=1}^{n} a_{ij}^u \bar{x}_j \leq b_i^l \leq b_i.$$

If $\bar{x}$ is the false solution for *Model (18)*, then

$$\sum_{j=1}^{n} a_{ij}\bar{x}_j \leq b_i \ \rightarrow c\bar{x} \ \leq \ z^*,$$

$$x_j \geq 0,$$

$$z^l \ = \sum_{j=1}^{n} c_j^l \bar{x}_j \leq \sum_{j=1}^{n} c_j \bar{x}_j \leq \ z^*.$$

In the same way, it can be proved for z^u *Model (17)* as above.

3 | Fuzzy Cost, Revenue and Profit Efficiency

The cost, revenue and profit efficiency of DMUs with fuzzy data in the form of two semi-fuzzy and full-fuzzy modes can be found in the following conditions.

I. The inputs/outputs are fuzzy numbers, and the price vector is crisp.

II. The inputs/outputs are crisp, and the price vector is fuzzy.

III. The inputs/outputs and the price vector are fuzzy.

The model presented based on the α-cut method finds the upper and lower bounds of efficiencies in an interval, and the membership function for efficiencies is defined as the inverse of the lower and upper boundaries. So the cost. revenue and profit efficiency score is a fuzzy number, and it changes in the interval

[0,1]. On the other hand, if the upper and lower bounds are inverse to α. Fuzzy data can be converted into crisp numbers with the α-cut approach where an upper and lower bound for efficiency score in an interval is obtained in an interval [8]. The fuzzy membership function can be obtained by plotting numbers and interpolating them for different α. In this section, the assumption is based on the fact that in the full fuzzy state, the input and output data and the input and output prices are of the type of triangular numbers. Therefore, it can be written for cost efficiency [5], [13].

If

$$\tilde{x}_{ij} = (x_{ij}^L, x_{ij}^M, x_{ij}^U),$$

$$\tilde{y}_{rj} = (y_{rj}^L, y_{rj}^M, y_{rj}^U),$$

$$\tilde{c}_i = \left(c_i^L, c_i^M, c_i^U\right).$$

$$\widetilde{CE}_P = \text{Max} \sum_{r=1}^{s} u_r \tilde{y}_{rp,}$$

$$\text{s.t,}$$

$$\sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{i=1}^{m} v_i \tilde{x}_{ij} \leq 0, \qquad j = 1, \ldots, n, \tag{19}$$

$$v_i \leq \tilde{c}_i, \qquad i = 1, \ldots, m,$$

$$u_r \geq 0, \qquad r = 1, \ldots, s,$$

$$v_i \geq 0, \qquad i = 1, \ldots, m.$$

Using the α-cut, the lower and upper bounds of the fuzzy cost efficiency *Model (16)* are as follows.

$$(x_{ij})_\alpha = [(x_{ij})_\alpha^L, (x_{ij})_\alpha^U] = [\alpha x_{ij}^M + (1-\alpha)\, x_{ij}^L,\ \alpha x_{ij}^M + (1-\alpha)\, x_{ij}^U], \text{ for all } i, j,$$

$$(y_{rj})_\alpha = [(y_{rj})_\alpha^L, (y_{rj})_\alpha^U] = [\alpha y_{rj}^M + (1-\alpha)\, y_{ij}^L,\ \alpha y_{rj}^M + (1-\alpha)\, y_{rj}^U], \text{ for all } r, j,$$

$$(c_i)_\alpha = [(c_i)_\alpha^L, (c_i)_\alpha^U] = [\alpha c_i^M + (1-\alpha)\, c_i^L,\ \alpha c_i^M + (1-\alpha)\, c_i^U], \text{ for all } i.$$

$$(CE_P)_\alpha^L = \min \begin{cases} CE_P = \text{Max} \sum_{r=1}^{s} u_r y_{rp}, \\ \text{s.t,} \\ \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, & j = 1, \ldots, n, \\ v_i \leq c_i, & i = 1, \ldots, m, \\ u_r \geq 0, & r = 1, \ldots, s, \\ v_i \geq 0, & i = 1, \ldots, m. \end{cases} \tag{20}$$

$$(C_i)_\alpha^L \leq c_i \leq (C_i)_\alpha^U,$$

$$(x_{ij})_\alpha^L \leq x_{ij} \leq (x_{ij})_\alpha^U,$$

$$(y_{rj})_\alpha^L \leq y_{rj} \leq (y_{rj})_\alpha^U,$$

For all i,r.

$$(CE_P)^U_\alpha = \max \begin{cases} CE_P = \text{Max} \sum_{r=1}^{s} u_r\, y_{rp}, \\ \text{s.t,} \\ \sum_{r=1}^{s} u_r\, y_{rj} - \sum_{i=1}^{m} v_i\, x_{ij} \le 0, & j = 1, \dots, n, \\ v_i \le c_{i,} & i = 1, \dots, m, \\ u_r \ge 0, & r = 1, \dots, s, \\ v_i \ge 0, & i = 1, \dots, m, \end{cases} \tag{21}$$

$$(C_i)^L_\alpha \le c_i \le (C_i)^U_\alpha,$$
$$(x_{ij})^L_\alpha \le x_{ij} \le (x_{ij})^U_\alpha,$$
$$\left(y_{rj}\right)^L_\alpha \le y_{rj} \le \left(y_{rj}\right)^U_\alpha,$$
$$\text{For all } i, r.$$

According to the *Models (17)* and *(18),*

$$\begin{aligned} &(CE_P)^L_\alpha = \text{Max} \sum_{r=1}^{s} u_r (y_{rp})^L_\alpha, \\ &\text{s.t,} \\ &\sum_{r=1}^{s} u_r\, (y_{rp})^L_\alpha - \sum_{i=1}^{m} v_i \left(x_{ip}\right)^U_\alpha \le 0, \\ &\sum_{r=1}^{s} u_r \left(y_{rj}\right)^U_\alpha - \sum_{i=1}^{m} v_i \left(x_{ij}\right)^L_\alpha \le 0, \qquad \text{for all } \; j, j \ne p, \\ &v_i \le (c_i)^L_\alpha, \quad , \text{ for all } \; i, \\ &u_r \ge 0, \qquad , \text{ for all } \; r, \\ &v_i \ge 0, \qquad , \text{for all } \; i. \end{aligned} \tag{22}$$

$$\begin{aligned} &(CE_P)^U_\alpha = \text{Max} \textstyle\sum_{r=1}^{s} u_r (y_{rp})^U_\alpha, \\ &\text{s.t,} \\ &\sum_{r=1}^{s} u_r\, (y_{rp})^U_\alpha - \sum_{i=1}^{m} v_i \left(x_{ip}\right)^L_\alpha \le 0, \\ &\sum_{r=1}^{s} u_r \left(y_{rj}\right)^L_\alpha - \sum_{i=1}^{m} v_i \left(x_{ij}\right)^U_\alpha \le 0 \qquad \text{for all } \; j, j \ne p, \\ &v_i \le (c_i)^U_{\alpha,} \quad , \text{for all } \; i, \\ &u_r \ge 0, \qquad , \text{for all } \; r, \\ &v_i \ge 0, \qquad , \text{for all } \; i. \end{aligned} \tag{23}$$

In the following, inspired by *Models (19)* and *(20)*, we briefly generalise the described method for fuzzy revenue and profit efficiency.

$$\widetilde{RE}_P = \text{Min} \sum_{i=1}^{m} v_i\, \tilde{x}_{ip},$$

$$\text{s.t},$$

$$\sum_{r=1}^{s} u_r\, \tilde{y}_{rj} - \sum_{i=1}^{m} v_i\, \tilde{x}_{ij} \le 0, \qquad j = 1, \dots, n, \tag{24}$$

$$u_r \ge \tilde{p}_r, \quad r = 1, \dots, s,$$

$$u_r \ge 0, \quad r = 1, \dots, s,$$

$$v_i \ge 0, \quad i = 1, \dots, m.$$

Using α-cut, we write the lower and upper bounds of fuzzy revenue efficiency as follows:

$$(x_{ij})_\alpha = [(x_{ij})_\alpha^L, (x_{ij})_\alpha^U] = [\alpha x_{ij}^M + (1-\alpha)\, x_{ij}^L,\ \alpha x_{ij}^M + (1-\alpha)\, x_{ij}^U], \text{ for all } i, j,$$

$$(y_{rj})_\alpha = [(y_{rj})_\alpha^L, (y_{rj})_\alpha^U] = [\alpha y_{rj}^M + (1-\alpha)\, y_{ij}^L,\ \alpha y_{rj}^M + (1-\alpha)\, y_{rj}^U], \text{ for all } r, j,$$

$$(p_r)_\alpha = [(p_r)_\alpha^L, (p_r)_\alpha^U] = [\alpha p_r^M + (1-\alpha)\, p_r^L,\ \alpha p_r^M + (1-\alpha)\, p_r^U], \text{ for all } r.$$

$$(RE_P)_\alpha^L = \text{Min} \sum_{i=1}^{m} v_i (x_{ip})_\alpha^L,$$

$$\text{s.t},$$

$$\sum_{r=1}^{s} u_r\, (y_{rp})_\alpha^U - \sum_{i=1}^{m} v_i\, (x_{ip})_\alpha^L \le 0,$$

$$\sum_{r=1}^{s} u_r\, (y_{rj})_\alpha^L - \sum_{i=1}^{m} v_i\, (x_{ij})_\alpha^U \le 0, \quad \text{for all } j, j \ne p, \tag{25}$$

$$u_r \ge (p_r)_\alpha^U, \quad \text{for all } r,$$

$$u_r \ge 0, \quad \text{for all } r,$$

$$v_i \ge 0, \quad \text{for all } i.$$

$$(RE_P)_\alpha^U = \text{Min} \textstyle\sum_{i=1}^{m} v_i (x_{ip})_\alpha^U,$$

$$\text{s.t},$$

$$\textstyle\sum_{r=1}^{s} u_r\, (y_{rp})_\alpha^L - \sum_{i=1}^{m} v_i\, (x_{ip})_\alpha^U \le 0,$$

$$\sum_{r=1}^{s} u_r\, (y_{rj})_\alpha^U - \sum_{i=1}^{m} v_i\, (x_{ij})_\alpha^L \le 0, \qquad \text{for all } j, j \ne p, \tag{26}$$

$$u_r \ge (p_r)_\alpha^L, \quad \text{, for all } r,$$

$$u_r \ge 0, \quad \text{, for all } r,$$

$$v_i \ge 0, \quad \text{, for all } i.$$

Also, about profit efficiency

$$\widetilde{PE}_P = \text{Min} \sum_{i=1}^{m} v_i' \, \tilde{x}_{ip} - \sum_{r=1}^{s} u_r' \, \tilde{y}_{rp},$$

$$\begin{aligned}
&\text{s.t,}\\
&\sum_{r=1}^{s} u_r \, \tilde{y}_{rj} - \sum_{i=1}^{m} v_i \, \tilde{x}_{ij} \le 0, \qquad j = 1, \dots, n,\\
&u_r - u_r' \ge \tilde{p}_r, \quad r = 1, \dots, s,\\
&v_i' - v_i \ge -\tilde{c}_i, \quad i = 1, \dots, m,\\
&u_r' \ge 0, \quad r = 1, \dots, s,\\
&v_i' \ge 0, \quad i = 1, \dots, m,\\
&u_r \ge 0, \quad r = 1, \dots, s,\\
&v_i \ge 0, \quad i = 1, \dots, m.
\end{aligned} \tag{27}$$

$$(x_{ij})_\alpha = [(x_{ij})_\alpha^L \ , \ (x_{ij})_\alpha^U] = [\alpha x_{ij}^M + (1-\alpha)\, x_{ij}^L, \ \alpha x_{ij}^M + (1-\alpha)\, x_{ij}^U], \quad \text{for all } i, j,$$

$$(y_{rj})_\alpha = [(y_{rj})_\alpha^L, (y_{rj})_\alpha^U] = [\alpha y_{rj}^M + (1-\alpha)\, y_{ij}^L, \ \alpha y_{rj}^M + (1-\alpha)\, y_{rj}^U], \text{ for all } r, j,$$

$$(c_i)_\alpha = [(c_i)_\alpha^L \ , \ (c_i)_\alpha^U] = [\alpha c_i^M + (1-\alpha)\, c_i^L, \ \alpha c_i^M + (1-\alpha)\, c_i^U], \quad \text{for all } i,$$

$$(p_r)_\alpha = [(p_r)_\alpha^L \ , \ (p_r)_\alpha^U] = [\alpha p_r^M + (1-\alpha)\, p_r^L, \ \alpha p_r^M + (1-\alpha)\, p_r^U], \quad \text{for all } r.$$

$$(PE_P)_\alpha^L = \text{Min} \sum_{i=1}^{m} v_i' (x_{ip})_\alpha^U - \sum_{r=1}^{s} u_r' (y_{rp})_\alpha^L,$$

$$\begin{aligned}
&\text{s.t,}\\
&\textstyle\sum_{r=1}^{s} u_r \, (y_{rp})_\alpha^L - \sum_{i=1}^{m} v_i \, (x_{ip})_\alpha^U \le 0,\\
&\textstyle\sum_{r=1}^{s} u_r \, (y_{rj})_\alpha^U - \sum_{i=1}^{m} v_i \, (x_{ij})_\alpha^L \le 0, \qquad \text{for all } j, j \ne p,\\
&u_r - u_r' \ge (p_r)_\alpha^U, \quad \text{, for all } r,\\
&v_i - v_I' \le (c_i)_\alpha^L, \quad \text{, for all } i,\\
&v_i \ge 0, \quad \text{, for all } i,\\
&u_r \ge 0, \quad \text{, for all } r,\\
&u_r' \ge 0, \quad \text{, for all } r,\\
&v_I' \ge 0, \quad \text{, for all } i.
\end{aligned} \tag{28}$$

$$
\begin{aligned}
&(PE_P)^U_\alpha = \operatorname{Min} \sum_{i=1}^{m} v'_i (x_{ip})^L_\alpha - \sum_{r=1}^{s} u'_r (y_{rp})^U_\alpha,\\
&\text{s.t,}\\
&\textstyle\sum_{r=1}^{s} u_r \, (y_{rp})^U_\alpha - \sum_{i=1}^{m} v_i \, (x_{ip})^L_\alpha \leq 0,\\
&\textstyle\sum_{r=1}^{s} u_r \, (y_{rj})^L_\alpha - \sum_{i=1}^{m} v_i \, (x_{ij})^U_\alpha \leq 0, \quad \text{for all } j, j \neq p,\\
&u_r - u'_r \geq (p_r)^L_\alpha, \quad \text{, for all } r,\\
&v_i - v'_I \leq (c_i)^U_\alpha, \quad \text{, for all } i,\\
&v_i \geq 0, \quad \text{, for all } i,\\
&u'_r \geq 0, \quad \text{, for all } r,\\
&v'_I \geq 0, \quad \text{, for all } i.
\end{aligned} \tag{29}
$$

4 | Application of Cost, Revenue and Profit Efficiency in Downstream Petrochemical Industries

In this section, the calculation results obtained from the efficient evaluation of the cost, revenue and profit efficiency for the fuzzy data of corrugated double-wall pipes from the downstream petrochemical industries are presented so that each of the 7 production lines is considered as a DMU. First, for a better understanding of the model, the structure of the form of input and output variables is shown below.

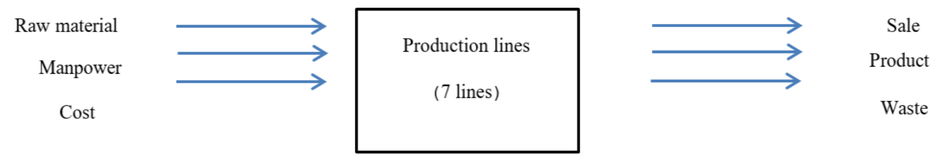

Fig. 1. Schematic view of the model.

Table 1. Definition of model variables.

Definition	Level	Variables
The purchase price of machinery and installation	Infrastructure	Machinery costs
The purchase price of land	Infrastructure	Land
Required physical space for machinery installation	Infrastructure	Physical space
Nominal production capacity of the machinery	Infrastructure	Capacity
Required physical space for production storage	Infrastructure	Store
Required raw material for the production of each product line	Production	Raw material
Required personnel for each production line	Production	Workforce
Fixed cost	Production	Salary
Variable cost	Production	Overtime
Variable cost	Production	Tools and consumable parts
Variable cost	Production	Energy
Variable cost	Production	Cost of maintenance
Variable cost	Production	Advertising and marketing
Fixed cost	Production	Depreciation cost
Fixed cost	Production	Other costs
Actual production rate of a DMU during the year	Production	Production
Actual sales rate of a DMU during the year	Production	Sale
Amount of waste per DMU during the year	Production	Wastage
Amount of recycling from waste the waste of one DMU during the year	Recycle	Recycle
Amount of waste that cannot be recycled	Recycle	Waste

Table 2. Fuzzy data table in 1403.

7	6	5	4	3	2	1	Variables
(3.5,4,4.2)	(4.3,5.1,5.3)	(15.8,16.7,17)	(15.17,16.8,17)	(6.48,8.35,8.5)	(9.7,10.3,10.5)	(3.2,4.6,5)	Salary (Million rial)
(0.52,0.58,0.6)	(0.85,0.92,0.95)	(2.88,3.1,3.5)	(2.85,3.1,3.5)	(1.3,1.6,2)	(0.18,0.22,0.25)	(0.62,0.66,.7)	Overtime (Million rial)
(0.37,0.4,0.45)	(0.59,0.65,0.7)	(2.87,2.95,3)	(2.1,2.5,3)	(0.936,1.2,1.5)	(1.32,1.36,1.4)	(0.45,0.48,0.5)	Energy (Million rial)
(0.945,1.13,1.2)	(1.59,1.65,1.8)	(5.37,5.9,6)	(5.31,5.5,5.75)	(2.4,2.8,3.2)	(3.39,3.6,3.85)	(1.15,1.5,1.8)	Tools and consumable parts (Million rial)
(0.32,0.45,0.5)	(0.53,0.65,0.7)	(1.79,1.85,2)	(1.76,1.9,2.1)	(0.81,0.9,0.95)	(1.13,1.52,1.6)	(0.388,0.45,0.5)	Cost of maintenance (Million rials)
(0.63,0.66,0.7)	(1.05,1.15,1.2)	(3.55,3.9,4.2)	(3.5,3.8,4)	(1.59,1.61,1.65)	(2.25,2.5,2.7)	(0.76,0.8,0.85)	Depreciation cost (Million rial)
(0.41,0.5,0.55)	(0.68,0.7,0.72)	(2.3,2.5,2.9)	(2.26,2.5,2.7)	(1.03,1.1,1.5)	(1.45,1.53,1.6)	(0.4,0.45,0.5)	Advertising and marketing (Million rials)
(1.12,1.5,1.8)	(1.9,2.2,2.5)	(6.4,6.6,7.1)	(6.3,6.5,7)	(2.9,3.3,3.5)	(3.6,4.1,4.5)	(1.25,1.3,1.35)	Other costs (Million rial)
(59.7,63.4,65.6)	(97.4,100.2,102.5)	(319.3,322.4,325.2)	(313.5,315.5,318.5)	(146,148.5,150)	(122.3,124.5,125.8)	(72.2,75.8,78.4)	Raw material (Million rial)
(4,6,9)	(7,10,12)	(7,10,12)	(7,10,12)	(5,7,9)	(4,6,8)	(3,5,7)	Workforce (Person)
(26.97,32.5,34)	(44.5,46.7,47)	(149.3,153.5,158)	(151.2,153.3,155)	(68.8,70.2,72)	(96.65,98.6,100)	(31.5,33.2,35.5)	Wastage (Ton)
(410,450,470)	(688.5,700,720)	(2325,2327,2400)	(2160,2200,2220)	(1044.4,1046.5,1048)	(1460.5,1463.5,1464)	(531.2,533.5,540)	Production (Ton)
(94.53,101.54,103)	(189.6,192.5,195)	(636.3,651,670)	(644,660,680)	(288.3,300,320)	(390.9,401.5,403)	(157.5,159.5,161)	Selling (Billion rial)
(28.4,29.7,31)	(46.8,47.5,49)	(147.7,149.5,152)	(149.6,154.5,156)	(68,70,72)	(95.6,99.5,100)	(25.3,27.2,28)	Recycle (Ton)
(0.202,0.25,0.322)	(0.35,0.452,0.5)	(1.05,1.156,1.28)	(1.15,1.26,1.3)	(0.49,0.53,0.55)	(0.702,0.73,0.75)	(035,0.42,0.45)	Waste (Ton)

Table 3. Data table with different α-cut in 1403 (If $\alpha = 0.1$).

7	6	5	4	3	2	1	Variables
[3.55,4.18]	[4.38,5.28]	[15.89,16.97]	[15.33,16.95]	[6.667,8.485]	[9.76,10.48]	[3.34,4.96]	Salary (Million rial)
[0.526,0.598]	[0.857,0.947]	[2.922,3.48]	[5.665,6.25]	[1.33,1.96]	[0.184,0.247]	[0.624,0.696]	Overtime (Million rial)
[0.373,0.445]	[0.596,0.695]	[2.878,2.995]	[2.14,2.95]	[0.9624,1.47]	[1.324,1.396]	[0.453,0.498]	Energy (Million rial)
[0.9635,1.193]	[1.596,1.785]	[5.423,5.99]	[5.329,5.725]	[2.44,3.16]	[3.411,3.825]	[1.185,1.77]	Tools and consumable parts (Million rial)
[0.333,0.513]	[0.542,0.695]	[1.796,1.985]	[1.774,2.08]	[0.819,0.945]	[1.169,1.592]	[0.394,0.495]	Cost of maintenance (Million rials)
[0.633,0.696]	[1.06,1.213]	[3.585,4.17]	[3.53,3.98]	[1.592,1.646]	[2.275,2.68]	[0.764, 0.845]	Depreciation cost (Million rial)
[0.419,0.545]	[0.682,0.718]	[2.32,2.86]	[2.284,2.68]	[1.037,1.46]	[1.458,1.595]	[0.405,0.495]	Advertising and marketing (Million rials)
[1.158,1.77]	[1.93,2.47]	[6.42,7.05]	[6.32,6.95]	[2.94,3.48]	[3.65,4.46	[1.255,1.345]	Other costs (Million rial)
[60.07,65.38]	[97.69,102.28]	[319.62,324.93]	[313.7,318.2]	[146.2,149.8]	[122.52,125.67]	[72.56,78.14]	Raw material (Million rial)
[4.24,8.74]	[7.3,11.8]	[7.3,11.8]	[7.3,11.8]	[5.2,8.8]	[4.2,7.8]	[3.2,6.8]	Workforce (Person)
[27.523,33.85]	[44.72,46.97]	[149.72,157.55]	[151.41,154.83]	[68.92,71.802]	[96.845,99.86]	[31.68,35.28]	Wastage (Ton)
[414,468]	[689.65,718]	[2325.2,2392.7]	[2164,2218]	[1044.61,1047.8]	[1460.8,1463.95]	[531.43,539.35]	Production (Ton)
[95.231,102.854]	[189.89,194.75]	[637.77,668.1]	[645.6,678]	[289.47,318]	[355.96,366.85]	[157.7,160.85]	Selling (Billion rial)
[28.53,30.87]	[46.87,48.858]	[147.88,151.75]	[150.09,155.85]	[68.2,71.8]	[95.99,99.95]	[25.49,27.92]	Recycle (Ton)
[0.207,0.315]	[0.360,0.495]	[1.06,1.268]	[1.161,1.296]	[0.494,0.548]	[0.7048,0.748]	[0.492,0.447]	Waste (Ton)

Next, in the same way, let's get α-cut for 0.7, 0.5, 0.3 and 1.

5 | Finding

As explained in the explanation of cost, revenue and profit efficiency models. One of the necessary conditions for calculating the aforementioned efficiency is to have the price of inputs and outputs (p_r , c_i). In the above example, the type of inputs is often the type of cost or production inputs. All of them have a price and are practically equal to 1, So there is no need for pricing. In relation to outputs, the sales amount is in rials and for production and production waste, p_r is between [580000,610000], which is the forecast range of the selling price of goods. According to this data, we will have coding by GAMZ software.

Table 4. The results of the evaluation of the efficiency of DMUs with different α-cuts.

DMU	1	2	3	4	5	6	7
$(CEp)_{0.1}$	[0.965,0.97]	[0.601,0619]	[0.904,0.939]	[0.895,0901]	[0.699,0.749]	[0.546,0.557]	[0.901,0.93]
$(REp)_{0.1}$	[0.526,0.731]	[0.782,0.833]	[0.381,0.541]	[0.544,0555]	[0.734,0.743]	[0.566,0.802]	[0.522,0.917]
$(PEp)_{0.1}$	[0.904,0.923]	[0.959,0.968]	[0.696,0.708]	[0.74,0.763]	[0.778,0.804]	[0.924,0.961]	[0.754,0.865]
$(CEp)_{0.3}$	[0.964,0.969]	[0.572,0.585]	[0.908,0.938]	[0.897,0.9013]	[0.702,0.739]	[0.547,0.556]	[0.891,0.91]
$(REp)_{0.3}$	[0.526,0.528]	[0.818,0.859]	[0.525,0.539]	[0.545,0.549]	[0.734,0.739]	[0.59,0.791]	[0.522,0.523]
$(PEp)_{0.3}$	[0.913,0.915]	[0.924,0.93]	[0.6995,0.723]	[0.743,0.761]	[0.78,0798]	[0.92,0.957]	[0.768,0.86]
$(CEp)_{0.5}$	[0.966,0.968]	[0554,0.551]	[0.912,0.935]	[0897,0.905]	[0.706,0.730]	[0.549,0.555]	[0.892,0.900]
$(REp)_{0.5}$	[0.526,0.527]	[0.856,0.887]	[0.525,0.537]	[0.546,0.549]	[0.735,0.740]	[0.616,0.777]	[0.522,0.523]
$(PEp)_{0.5}$	[0.906,0.917]	[0.888,0.89]	[0.7,0.72]	[0.745,0.758]	[0.787,0.794]	[0.929,0.952]	[0.798,0.853]
$(CEp)_{0.7}$	[0.966,0.968]	[0.624,0.630]	[0.916,0.929]	[0.898,0.900]	[071,0.722]	[0.55,0.553]	[0.894,0.899]
$(REp)_{0.7}$	[0.526,0.527]	[0.783,0.799]	[0.527,0.535]	[0.546,0.548]	[0.735,0.737]	[0.646,0.742]	[0.522,0.523]
$(PEp)_{0.7}$	[0.908,0.9145]	[0.982,0.984]	[0.701,0.717]	[0.748,0.756]	[0.781,0.789]	[0.934,0.948]	[0.814,0.846]
$(CEp)_{1}$	[0.964,0.97]	[0.615,0636]	[0.902,0.939]	[0.896,0.903]	[0.697,0.752]	[0.546,0558]	[0.888,0.904]
$(REp)_{1}$	[0.525,0.528]	[0.766,0.87]	[0.525,0.542]	[0.544,0.55]	[0733,0.741]	[0.565,0.806]	[0.522,0.57]
$(PEp)_{1}$	[0.902,0.915]	[0.975,0.986]	[0698,0.727]	[0.738,0763]	[0.777,0.806]	[0.921,0.963]	[0.761,0.87]

As it is evident from the table above, DMU1 has the highest cost efficiency compared to other DMUs with different α-cut. This means the optimal management of cost control in order to reduce the total price of the product, which ultimately leads to an increase in the operating profit of that unit. Also, DMU2 has the highest revenue efficiency among other DMUs which also means applying proper management to increase production and profitability. But in terms of profit efficiency, in most α-cuts, DMU2 has the highest score, which means correct management of both inputs and outputs of a system. Now. For example, if we get the membership function of DMUs of one of the inputs, which are all triangular fuzzy numbers. Let's get its α-cuts; for example, we will have the raw material input variable.

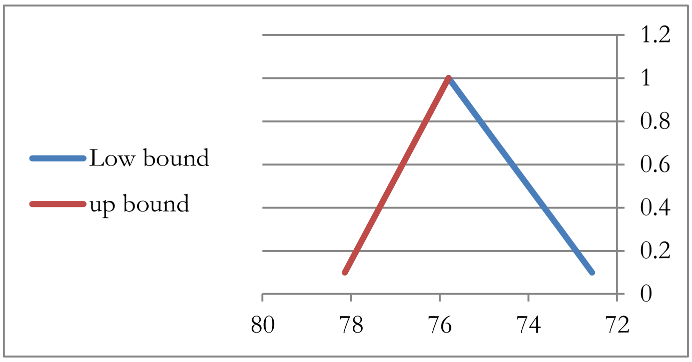

Fig. 2. Shape of the membership function of the raw material variable for different α-cuts.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \leq 72.56, \\ \dfrac{x-75.8}{3.24}, & 72.56 \leq x \leq 75.8, \\ \dfrac{x-78.14}{2.34}, & 75.8 \leq x \leq 78.14, \end{cases}$$

6 | Conclusion

Researchers have done many studies in relation to calculating the cost, revenue and profit efficiency, especially the cost efficiency with fuzzy data. But no work has been done to solve its dual form except for the cost efficiency score. Therefore, in the present research. By generalising it to calculate the fuzzy revenue and profit efficiency, the solution of the practical example in the downstream petrochemical industries has been addressed using α-cut. Then, the results are in the form of intervals and are ranked based on their efficiency scores for DMUs. De-fuzzification method can also be used to solve efficiency problems, which cannot be relied upon due to the high inaccuracy of the answers and high error.

Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data Availability

The datasets used and/or analyzed during the current study are available from the corresponding author upon reasonable request.

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