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A New Approach Utilizing Addition-Min Composition in a Two-Sided Fuzzy Relation

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Abstract

This study focuses on the bilateral requirements of terminals within a Peer-To-Peer (P2P) network system, specifically examining two-sided fuzzy relation inequalities using addition-min composition. Each solution derived from this two-sided fuzzy relation system represents a viable flow control strategy for the associated P2P network. The main topics covered include 1) identifying a minimal solution that is less than or equal to a specified solution, 2) identifying a maximal solution that is greater than or equal to a specified solution, and 3) outlining the structure of the solution set for the fuzzy relation system. The goals of 1) and 2) are to pinpoint particular minimal or maximal solutions within the two-sided system. We introduce two algorithms, Algorithm I and II, to determine these specific minimal and maximal solutions with polynomial computational complexities. Their effectiveness is demonstrated through various numerical examples. It is observed that all minimal and maximal solutions can entirely characterize the complete solution set for the two-sided system, and it may also be non-convex.

Keywords: Addition-min composition, Fuzzy relation inequality, Maximal solutions, Two-sided.

1 | Introduction

Sanchez [1] is credited with the initial introduction of the fuzzy relation system, encompassing equations and inequalities. The most commonly examined fuzzy relation system involves operations such as max-min or max-product. Research on fuzzy relation systems primarily focuses on two key areas: 1) determining the solution set, and 2) addressing the associated optimization problem [2], [3]. proposed a novel form of fuzzy relation system characterized by addition-min composition [4]. This addition-min Fuzzy Relation Inequality

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(FRI) was developed to model P2P network systems. In instances where the download traffic requirements of terminals within such a system are equal to or greater than $b_1, b_2, ..., b_m$, and so forth, the P2P network system can be represented as the following FRI system.

$$\begin{cases} c_{11} \wedge y_1 + c_{12} \wedge y_2 + ... + c_{1n} \wedge y_n \geq b_1, \\ c_{21} \wedge y_1 + c_{22} \wedge y_2 + ... + c_{2n} \wedge y_n \geq b_2, \\ .... \\ c_{m1} \wedge y_1 + c_{m2} \wedge y_2 + ... + c_{mn} \wedge y_n \geq b_m. \end{cases} \quad (1)$$

The initial research on addition-min fuzzy relation systems concentrated on identifying *Model (1)* minimal solutions [4]. The authors established a necessary and sufficient condition to determine if a general solution is minimal within the entire solution set. Additionally, they introduced an effective method for finding certain specific minimal solutions of *Model (1)* based on this examination condition.

To facilitate understanding, we will revisit using an addition-min fuzzy relation inequalities system within the P2P file-sharing network (refer to *Fig. 1*). We represent all terminals in the P2P network $T_1, T_2, ..., T_n$, each pair of terminals connected by a line. The bandwidth between terminals T_i T_j is denoted as c_{ij} (when data is being sent from T_j to T_i). Let y_j's represent the quality level at which T_j each terminal shares its local data with other terminals. Due to bandwidth limitations, T_i the actual download T_j traffic is illustrated when a terminal download $c_{ij} \wedge y_j$ s the resources it has requested.

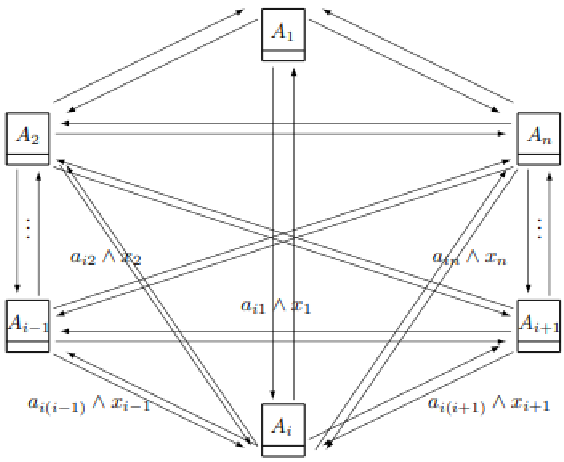

Fig. 1. P2P file sharing system.

T_i We will choose a terminal with the greatest download traffic to obtain the resources it needs. If T_i the download traffic requirement is at least b_i, then we have

$$c_{i1} \wedge y_1 + c_{i2} \wedge y_2 + ... + c_{in} \wedge y_n \geq b_i.$$

By integrating the needs of all terminals, we create the one-sided *Model (1)*, which has been examined in previous studies [5], [6].

In the current literature [7], [8] the terminals' needs have been addressed one-sidedly. Specifically, the authors focused solely on the minimum requirements, neglecting the maximum limits. To address this oversight, we expand on the upper limits of the requirements in this study. We assume that the bilateral requirements for the terminal T_i are at least b_i and at most d_i. With this approach, the requirements for all terminals can be represented as a two-sided FRI system using addition-min composition.

$$\begin{cases} b_1 \le c_{11} \wedge y_1 + c_{12} \wedge y_2 + \ldots + c_{1n} \wedge y_n \le d_1, \\ b_2 \le c_{21} \wedge y_1 + c_{22} \wedge y_2 + \ldots + c_{2n} \wedge y_n \le d_2, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ b_m \le c_{m1} \wedge y_1 + c_{m2} \wedge y_2 + \ldots + c_{mn} \wedge y_n \le d_m. \end{cases}$$

Our work includes several key contributions: 1) identifying a minimal solution that is less than or equal to a specified solution in the two-sided *Model (2)*, 2) identifying a maximal solution greater than or equal to a specified solution in the two-sided *Model (2)*, and 3) Using the findings from 1 and 2 to develop the structure of the solution set for *Model (2)* and providing a formal proof.

The remainder of this document is structured as follows. Section 2 presents essential notations and definitions. Section 3 introduces an efficient method for finding a minimal solution that is less than or equal to a specified solution. Section 4 addresses finding a maximal solution greater than or equal to a given solution. Building on the findings from Sections 3 and 4, Section 5 explores the structure of the solution set for *Model (2)*. Finally, Sections 6 and 7 contain the discussion and conclusion, respectively.

2 | Preliminaries

This part clarifies certain notations and definitions related to the *Model (2)*. For ease of reference, we will define two index sets below. $I = \{1, 2, \ldots, m\}, J = \{1, 2, \ldots, n\}$.

All the parameters $\{c_{ij} | i \in I, j \in J\}$ and variables $\{y_j | j \in J\}$ are standardized. Once standardization is complete, we typically assume that $c_{ij}, y_j \in [0,1]$. *Model (2)* can be expressed in matrix form as follows:

$$b^T \le C o y^T \le d^T, \quad (3)$$

where

$$b = (b_1, b_2, \ldots, b_m), C = (c_{ij})_{m \times n}, y = (y_1, y_2, \ldots, y_n),$$
$$d = (d_1, d_2, \ldots, d_m),$$

Denote

$$S = \left\{ y \in [0,1]^n \middle| b^T \le C o y^T \le d^T \right\}. \quad (4)$$

The symbol S denotes the collection of all solutions for *Model (2).*

Definition 1. A *Model (1)* or *(2)* is considered consistent if it has at least one solution [9].

A solution $\breve{Y} \in S$ is said to be minimal, if $y \in S, y \le \breve{Y}$ any, which implies that $y = \breve{Y}$. On the contrary, a solution $\hat{Y} \in S$ is said to be maximal, if for any $y \in S, y \ge \hat{Y}$, means that $y = \hat{Y}$. Besides, a solution $\hat{Y} \in S$ is considered maximum if $y \le \hat{Y}$ it holds for any. $y \in S$.

Proposition 1. If *Model (1)* is consistent, it possesses a singular maximum solution [4].

Proposition 2. If the *Model (1)* is consistent, the solution set can be expressed as $\bigcup_{\breve{x} \in \breve{X}} [\breve{x}, \hat{x}]$ where $\hat{x}$ denotes its unique maximum solution and $\breve{X}$ is the collection of all minimal solutions [9].

Proposition 3. If *Model (1)* has at least two minimal solutions, it will have an infinite number of minimal solutions [9].

Proposition 4. If x^0 a *Model (1)*, then there exists a minimal solution $\breve{x}^0$, where $\breve{x}^0 \le x^0$ [10].

In Sections 3 and 4, we consistently consider $y^0 \in S$ a specified solution to *Model (2)*. Furthermore, we will conduct a deeper analysis of the structure of the solution set for *Model (2)*, taking into account all minimal and maximal solutions.

3 | Identifying a Minimal Solution

Let

$$I_1 = \left\{ i \in I \mid \sum_{2 \le j \le n} c_{ij} \wedge y_j^0 < b_i \right\}. \quad \textbf{(5)}$$

If $I_1 \neq \varnothing$ we refer to

$$\Delta_i^1 = b_i - \sum_{2 \le j \le n} c_{ij} \wedge y_j^0, \text{for all } i \in I_1. \quad \textbf{(5)}$$

It is clear that $\Delta_i^1 > 0$ for any $i \in I_1$. Denote

$$\breve{y}_1 = \begin{cases} 0 & \text{if } I_1 = \varnothing, \\ \bigvee_{i \in I_1} \Delta_i^1 & \text{if } I_1 \neq \varnothing, \end{cases} \quad \textbf{(5)}$$

and $\breve{y}_1 = \left(\breve{y}_1, y_2^0, ..., y_n^0 \right)$.

Remark 1. We have $y_1^0 \ge \breve{y}_1 \ge 0$ and $y^0 \ge \breve{Y}^1 \in S$. Moreover $y_1' < \breve{y}_1$, it holds that.

$$\left(y_1', y_2^0, y_3^0, ..., y_n^0 \right) \notin S. \quad \textbf{(5)}$$

Proof: If $I_1 = \varnothing$ so, then clearly $y^0 \ge \breve{Y}^1 \in S$. Now, take into account the situation $I_1 \neq \varnothing$. There exists $i_1 \in I_1$, such that

$$0 < \breve{y}_1 = \Delta_{i1}^1 = b_{i1} - \sum_{2 \le j \le n} c_{i_1 j} \wedge y_j^0 \le c_{i_1 1} \wedge y_1^0 \le y_1^0.$$

This also suggests $\breve{y}_1 \le c_{i_1 1}$ $c_{i_1 1} \wedge \breve{y}_1 = \breve{y}_1$ that. Specifically, $y_1' < \breve{y}_1$ we have

$$c_{i_1 1} \wedge y_1' = y_1' < \breve{y}_1 = b_{i1} - \sum_{2 \le j \le n} c_{i_1 j} \wedge y_j^0,$$

and thus

$$\left(y_1', y_2^0, y_3^0, ..., y_n^0 \right) \notin S,\ i \notin I_1,$$

we have

$$c_{i1} \wedge \breve{y}_1 + \sum_{2 \le j \le n} c_{ij} \wedge y_j^0 \ge \sum_{2 \le j \le n} c_{ij} \wedge y_j^0 \ge b_i.$$

For $i \in I_1$, we have

$$c_{i1} + \sum_{2 \le j \le n} c_{ij} \wedge y_j^0 \ge \sum_{2 \le j \le n} c_{ij} \wedge y_j^0 \ge b_i,$$

and

$$\breve{y}_1 + \sum_{2 \le j \le n} c_{ij} \wedge y_j^0 \ge \Delta_i^1 + \sum_{2 \le j \le n} c_{ij} \wedge y_j^0 = b_i,$$

and hence

$$c_{i1} \wedge \breve{y}_1 + \sum_{2\le j\le n} c_{ij} \wedge y_j^0 \ge b_i,$$

We have $y^0 \ge \breve{Y}^1 \in S$. We ultimately receive $(\breve{y}_1, \breve{y}_2, ..., \breve{y}_n) = \breve{Y} \in S$ such that

$$\breve{Y} = \breve{Y}^n \le ... \le \breve{Y}^2 \le \breve{Y}^1 \le y^0.$$

Remark 2. $\breve{Y}$ is minimal in S.

Proof: This is a straightforward conclusion based on *Remark 1*.

Theorem 1. Let $y^0 \in S$ be a solution of *Model (2)*. Then, a minimal solution $\breve{Y} \in S$ $\breve{Y} \le y^0$ exists [11].

Proof: It can be inferred from *Remarks 1* and *2*.

Algorithm 1. Solving a minimal solution less than or equal to y^0.

Step 1. Let $k := 1$.

Step 2. Compute

$$I_k = \left\{ i \in I \,\middle|\, \sum_{1\le j\le k-1} c_{ij} \wedge \breve{y}_j + \sum_{k+1\le j\le n} c_{ij} \wedge y_j^0 < b_i \right\}. \tag{10}$$

Step 3. If $I_k = \varnothing$, then $\breve{y}_k = 0$. Otherwise, if $I_k \ne \varnothing$ they compute,

$$\breve{y}_k = \vee_{i\in I_k} \Delta_i^k, \tag{11}$$

$$\Delta_i^k = b_i - \sum_{1\le j\le k-1} c_{ij} \wedge \breve{y}_j - \sum_{k+1\le j\le n} c_{ij} \wedge y_j^0. \tag{12}$$

Step 4. Let $k := 1$.

Step 5. If $k \le n$, then return to *Step 2*. Otherwise $k > n$, go to *Step 6*.

Step 6. $\breve{Y} = (\breve{y}_1, \breve{y}_2, ..., \breve{y}_n)$ is a minimal solution of *Model (2)*, satisfying $\breve{Y} \le y^0$.

Example 1. Consider a two-sided fuzzy relation inequalities system that utilizes addition-min composition, as outlined below.

$$\begin{cases} 1.0 \le 0.5 \wedge y_1 + 0.6 \wedge y_2 + 0.3 \wedge y_3 + 0.3 \wedge y_4 \le 1.6, \\ 1.1 \le 0.7 \wedge y_1 + 0.4 \wedge y_2 + 0.4 \wedge y_3 + 0.2 \wedge y_4 \le 1.8, \\ 1.3 \le 0.6 \wedge y_1 + 0.2 \wedge y_2 + 0.4 \wedge y_3 + 0.3 \wedge y_4 \le 1.7. \end{cases}$$

Let $y^0 = (0.6, 0.4, 0.5, 0.4)$ be a given solution of the system. Find a minimal solution to the system that is less than or equal to y^0.

Solution: For $k = 1$, since

$$\begin{cases} 0.6 \wedge y_2^0 + 0.3 \wedge y_3^0 + 0.3 \wedge y_4^0 = 1.0, \\ 0.4 \wedge y_2^0 + 0.4 \wedge y_3^0 + 0.2 \wedge y_4^0 = 1.0, \\ 0.2 \wedge y_2^0 + 0.4 \wedge y_3^0 + 0.3 \wedge y_4^0 = 0.9. \end{cases}$$

Thus

$$\breve{y}_1 = \vee_{i\in I_1} \Delta_i^1 = \Delta_2^1 \vee \Delta_3^1 = 0.1 \vee 0.4 = 0.4,$$

$$\breve{y}_2 = \vee_{i\in I_2} \Delta_i^2 = \Delta_2^2 \vee \Delta_3^2 = 0.1 \vee 0.2 = 0.2,$$

$$\breve{y}_3 = \vee_{i\in I_3} \Delta_i^3 = \Delta_1^3 \vee \Delta_2^3 \vee \Delta_3^3 = 0.1 \vee 0.3 \vee 0.4 = 0.4,$$

$$\breve{y}_4 = \vee_{i\in I_4} \Delta_i^4 = \Delta_1^4 \vee \Delta_2^4 \vee \Delta_3^4 = 0.1 \vee 0.1 \vee 0.3 = 0.3.$$

Consequently,

$$\breve{Y} = (\breve{y}_1, \breve{y}_2, \breve{y}_3, \breve{y}_4) = (0.4, 0.2, 0.4, 0.3).$$

It is a minimal solution to the system, meeting the condition $\breve{Y} \le y^0$.

4 | Finding a Maximal Solution More than or Equal to y^0

Let

$$I_1' = \left\{ i \in I \,\middle|\, c_{i1} + \sum_{2\le j\le n} c_{ij} \wedge y_j^0 > d_i \right\}. \quad \textbf{(13)}$$

If $I_1' \ne \varnothing$, we denote

$$\nabla_i^1 = d_i - \sum_{2\le j\le n} c_{ij} \wedge y_j^0, \text{for all } i \in I_1'. \quad \textbf{(14)}$$

It could be checked $0 \le \nabla_i^1 < c_{i1}$ for any $i \in I_1'$. Denote

$$\hat{y}_1 = \begin{cases} 1 & \text{if } I_1' = \varnothing, \\ \wedge_{i\in I_1'} \nabla_i^1 & \text{if } I_1' \ne \varnothing. \end{cases} \quad \textbf{(15)}$$

and

$$\hat{Y}^1 = (\hat{y}_1, {y^0}_2, \ldots, {y_n}^0). \quad \textbf{(16)}$$

Remark 3. $\hat{Y}$ is a maximal solution in S, with $\hat{Y} \ge y^0$.

Proof: Analogous to the demonstration provided in *Remark 1*.

Theorem 2. Let $y^0 \in S$ be a solution of *Model (2)*. Then, a maximal solution $\hat{Y} \in S$ $\hat{Y} \ge y^0$ exists [9].

Algorithm 2. solving a maximal solution more than or equal to y^0

Step 7. Let $k := 1$.

Step 8. Compute

$$I_k' = \left\{ i \in I \,\middle|\, \sum_{1\le j\le k-1} c_{ij} \wedge \hat{y}_j + c_{ik} + \sum_{k+1\le j\le n} c_{ij} \wedge y_j^0 > d_i \right\}. \quad \textbf{(17)}$$

Step 9. If $I_k \ne \varnothing$, then compute

$$\nabla_i^k = d_i - \sum_{1\le j\le k-1} c_{ij} \wedge \hat{y}_j - \sum_{k+1\le j\le n} c_{ij} \wedge y_j^0. \quad (18)$$

$i \in I_1'$ For any, go to *Step 9*. Otherwise, go to *Step 9* directly.

Step 10. Compute $\hat{y}_k$ as follows,

$$\hat{y}_k = \begin{cases} 1 & \text{if } I_k' = \varnothing, \\ \bigwedge_{i\in I_k'} \nabla_i^k & \text{if } I_k' \ne \varnothing. \end{cases} \quad (19)$$

Step 11. Let $k := 1$.

Step 12. If $k \le n$, then return to *Step 11*. Otherwise, if $k > n$ not, then go to *Step 13*.

Step 13. $\hat{Y} = (\hat{y}_1, \hat{y}_2, ..., \hat{y}_n)$ is a maximal solution of *Model (2)*, satisfying $\hat{Y} \ge y^0$.

Remark 4. Let $y^1, y^2 \in S$ be two solutions for *Model (2)* that are satisfying $y^1 \le y^2$. Then we have $[y^1, y^2] \subseteq S$.

Proof: Take arbitrary

$$y = (y_1, y_2, ..., y_n) \in [y^1, y^2].$$

Suppose

$$y^1 = (y_1^1, y_2^1, ..., y_n^1),\ y^2 = (y_1^2, y_2^2, ..., y_n^2),$$

and. It is clear that $y \in [0,1]^n$ and

$$y_j^1 \le y_j \le y_j^2, \text{for all } j \in J. \quad (20)$$

Since $y^1, y^2 \in S$ our *Model (2)*, we have

$$\begin{cases} b_i \le c_{i1} \wedge y_1^1 + c_{i2} \wedge y_2^1 + ... + c_{in} \wedge y_n^1 \le d_i, \\ b_i \le c_{i1} \wedge y_1^2 + c_{i2} \wedge y_2^2 + ... + c_{in} \wedge y_n^2 \le d_i. \end{cases}$$

Considering inequalities, we further get.

$$\begin{aligned} & b_i \le c_{i1} \wedge y_1^1 + c_{i2} \wedge y_2^1 + ... + c_{in} \wedge y_n^1, \\ & \le c_{i1} \wedge y_1 + c_{i2} \wedge y_2 + ... + c_{in} \wedge y_n, \\ & \le c_{i1} \wedge y_1^2 + c_{i2} \wedge y_2^2 + ... + c_{in} \wedge y_n^2 \le d_i. \end{aligned}$$

Hence, y is also a solution of *Model (2)*, and we have $[y^1, y^2] \subseteq S$.

Theorem 3. Suppose *Model (2)* is consistent [11]. Then its complete solution set, denoted by S, could be characterized as follows,

$$S = \bigcup_{\breve{Y}\in\breve{S}, \hat{Y}\in\hat{S}, \breve{Y}\le\hat{Y}} \left\{ y \middle| \breve{Y} \le y \le \hat{Y} \right\},$$

where $\breve{S}$ is the set of all minimal solutions, while $\breve{S}$ represents the set of all maximal solutions of *Model (2)*.

Proof: The subsequent observation is a straightforward extension of *Remark 4*.

Theorem 3 establishes that for a consistent *Model (2)*, the solution set can be fully characterized by its minimal and maximal solutions. Furthermore, this solution set can represent a collection of closed intervals. This structural representation of the solution set in *Model (2)* is analogous to a one-sided fuzzy relation inequalities system utilizing addition-min composition. However, an analysis of the convexity of the solution set reveals notable differences. Prior formal proofs have demonstrated that the solution set of a one-sided addition-min system is convex; however, this claim does not apply to the two-sided *Model (2)*.

For example, consider the following two-sided addition-min system,

$$\begin{cases} 0.5 \le 0.3 \wedge x_1 + 0.7 \wedge x_2 \le 0.9, \\ 0.6 \le 0.5 \wedge x_1 + 0.4 \wedge x_2 \le 0.8. \end{cases}$$

It could be easily checked that both $y^1 = (0.2,1)$, $y^2 = (0.4,0.6)$ are solutions to the system. Take the convex combination of y^1 and y^2 as,

$$y^c = 0.5 * y^1 + 0.5 * y^2 = (0.3,0.8).$$

Then we have

$$0.3 \wedge 0.3 + 0.7 \wedge 0.8 = 1 > 0.9.$$

Hence, the convex combination yc is no longer a solution of the system. This indicates that the system's solution set is a non-convex set.

5 | Conclusion

The second system discussed in this study is a two-sided FRI system with an addition-min composition. The research presented in this paper delves into various characteristics of this system, offering detailed methodologies for determining both minimal and maximal solutions. By establishing the existence of these solutions, the study proceeds to demonstrate a structural theorem governing the system's solution *Model (2)*. The findings presented here are anticipated to facilitate further investigations into addition-min fuzzy relation inequalities. Notably, it is revealed that the entire system *Model (2)* is entirely defined by its minimal and maximal solutions, which collectively form a union of closed intervals. Unlike the one-sided *Model (2)*, *Model (2)* may exhibit infinite maximal solutions and a non-convex solution set. A notable limitation of this research is the absence of a practical approach for determining the complete solution set of the *Model (2)*, with only specific solutions currently identified. Future research endeavors will resolve all system *Model (2)* while exploring optimization problems constrained by this system, which presents another intriguing avenue for investigation.

Conflict of Interest

The authors declare no conflict of interest.

Data Availability

All data are included in the text.

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