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Malmquist Productivity Index to a Two-Stage Structure in the Presence of Uncertain Data

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Abstract

Network Data Envelopment Analysis (NDEA) models assess the processes of the underlying system at a certain moment and disregard the dynamic effects inside the production process. Hence, distorted efficiency evaluation is gained that might give misleading information to Decision-Making Units (DMUs). However, the dynamic DEA model discusses the repetition of a single-period form over a long-term period, and it appears as a shape of a time series one that includes a particular construction in each period. Malmquist Productivity Index (MPI) assesses efficiency changes over time, which are measured as the product of recovery and frontier-shift terms, both coming from the DEA framework. In this study, a form of MPI involving network structure for evaluating DMUs in the presence of uncertainty and undesirable outputs in two periods of time is presented. To cope with the uncertainty, we use the stochastic p-robust approach, and the weak disposability of Kuosmanen [1] is utilized to take care of undesirable outputs. The proposed fractional models are linearized, applying the Charnes and Cooper transformation, and they are applied to evaluate the efficiency of 11 oilfields to identify the main factors determining their productivity, utilizing the data from the 2020 to 2021 period. The results show that the management of resource usage, especially forces and equipment, is inappropriate, and investment is not sufficient. This specific attribute highlights the necessity to enhance the rate of investment to substitute the depreciated funds.

Keywords: Data envelopment analysis, Stochastic p-robust, Network data envelopment analysis, Malmquist productivity index, Oilfields.

1 | Introduction

Since its beginning (see Charnes et al. [2]), Data Envelopment Analysis (DEA) has been applied to evaluate the efficiencies of a collection of Decision-Making Units (DMUs) in which estimating the efficiency frontier does not require the recognition of the production function. Classical DEA models consider each DMU as a

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black box, ignoring the internal relations of processes. However, in the real world, DMUs may contain several linked processes. Network Data Envelopment Analysis (NDEA) models, an extension of classical DEA models, are developed for the efficiency evaluation of DMUs, taking into consideration their internal relations via intermediate products in assessing efficiency. However, NDEA models disregard the dynamic effects within the production processes, which is the case of various real-world applications in [3]. Malmquist Productivity Index (MPI), presented by Malmquist [4], is a quality index for analyzing the consumption of production resources in different periods. The MPI not only defines patterns of productivity change and renders a new interpretation along with the managerial implication of each Malmquist component but also identifies strategic directions of an organization in past periods for proper choice in future periods. Also, it sounds like the MPI structure is a kind of series that has a particular structure in each period. *Table 1* summarizes some of the MPI-DEA developments and applications.

Table 1. Recent advances in applications in MPI-DEA/NDEA.

Authors	DEA Model	Type Factor	Applications Scope
Xu et al. [5]	CCR	Output	Airline companies
Li et al. [6]	SFA	Output	Forestry Company
Diwan [7]	Cobb–Douglas	Output/Input	Agricultural
Yang et al. [8]	CCR	Output	Regional eco-efficiency
Nedaei et al. [9]	CCR	Output	Oil and gas wells
Mahmoudi et al. [10]	SBM	Output/Input	Airline companies
Tone et al. [11]	CCR	Output	Insurance companies

However, in production processes, besides desirable outputs, there might be undesirable outputs whose decrease results in improved performance. For example, Pittman et al. [12] first studied the application of undesirable outputs to do an efficient assessment under the expanded model of Caves et al. [13] so that the efficiency of DMUs could be evaluated in the presence of desirable and undesirable outputs. Then, Tone [14] studied the efficiency of 12 Chinese commercial banks from 2005 to 2013 based on undesirable outputs and investigated the truth that considering undesirable outputs can make research results more reliable by comparing them with the results gained without considering undesirable output. *Table 2* summarizes some recent advances of undesirable output developments in MPI-DEA models.

Table 2. Recent advances of environmental factors in MPI- DEA/NDEA.

Authors	DEA Model	Environmental Factor	Applications Scope
Wang et al. [15]	CCR	Output	Industrial
Zhu et al. [16]	CCR	Output	Iron and Steel Industry
Li et al. [17]	SBM	Output/Input	Industrial systems
Tavana et al. [18]	CCR	Output	Banking industry
Bhardwaj et al. [19]	CCR	Output	Airline
Toloo et al. [20]	CCR	Output	Countries
Lee et al. [21]	SBM	Output/Input	Trunk streams
Asanimoghadam et al. [22]	ASBM	Output	Industrial airline
Salahi et al. [23]	ASBM	Output	Provinces in China
Shakouri et al. [24]	CCR	Output	Oil generation
Zhang et al. [25]	CCR	Output	Industrial system
Kuang et al. [26]	SBM	Output/Input	Public health center
Arabi et al. [27]	SBM	Output/Input	power plants

In all the above-mentioned research, inputs, and output parameters are considered to be exact, and the effect of uncertainty is ignored. Research detected that a small perturbation in the problem data may lead to critical variation in ranking. To treat uncertainty in the DEA models, various approaches, such as fuzzy programming, stochastic programming, and robust optimization, are used in the literature. *Table 3* summarizes the recent progress of the DEA models under uncertainty.

Table 3. Recent progress in stochastic, fuzzy, and robust optimization with MPI in the NDEA and DEA.

Authors	DEA/Uncertainty Parameters	Robust Approach	Applications Scope
Peykani et al. [28]	CCR	Fuzzy	Investment firms
Khaksar and Malakoutian [29]	CCR/BCC/Input	SFA	Banking sector
Salahi et al. [30]	CCR/CSW/ in-output	Interval	Energy/forest district
Salahi et al. [31]	Russell/ in-output	Interval	Banking sector
Salahi et al. [32]	CCR-CSW/ output	Bertsimas	Banking sector
Soltanzadeh et al. [33]	CCR/ in-output	Fuzzy	Airline companies
Akbarian et al. [34]	BCC	Interval	Numerical example
Shakouri et al. [24]	CCR/ input	Stochastic p-robust	Banking sector
Shakouri et al. [35]	CCR	Stochastic p-robust	Banking sector
Mehdizadeh et al. [36]	CCR	Stochastic	Commercial banks

Although the present literature has progressed significantly, all of the available DEA models consider either the pure undesirable outputs or uncertainties in problem data. So, in this paper, we present a combined model to measure the performance of DMUs with uncertain perspectives in the presence of undesirable outputs in dynamic settings. In the MPI framework, we apply the stochastic p-robust approach to attain robustness against the existing uncertainty for the CCR-DEA model. The stochastic approach searches to minimize the total expected cost among all scenarios. The optimal solution gained by applying it is probably very good for some scenarios but very poor for others. The weak disposable production technology of Kuosmanen [1] is employed for modeling undesirable outputs due to the weak disposability hypothesis, which is an essential development in computing the efficiency of DMUs with undesirable outputs compared with other technologies. The MPI not only reveals patterns of productivity change and presents a new interpretation along with the managerial implication of each Malmquist component but also recognizes the strategy shifts of exclusive companies under isoquant changes. We applied the proposed approach to the dataset of 11 oilfields from the 2020 to 2021 period to display the applicability of the model. Our contribution can be summarized as follows:

I. Assessing the efficiencies of an NDEA system and its internal processes over time by dynamic models simultaneously.

II. The MPI models are presented by considering desirable and undesirable outputs simultaneously.

III. Applying Kuosmanen's weakly disposable technology that is convex and more flexible with regard to the choice of non-uniform pollution abatement factors and preserving the linear structure.

IV. Describing the uncertainty in two optimistic and pessimistic scenarios, defining a robustness level for the MPI models that reflect the DMU's regress or progress.

V. The MPI computation is performed to evaluate the total efficiency of 11 oilfields in two time periods.

The remainder of this paper unfolds as follows: In the next section, a summary of a two-stage DEA model is given, and a brief review of weakly disposable technology and stochastic p-robust approach follows. In Section 3, by considering undesirable outputs and the p-robust approach, a model is proposed that calculates MPI under the NDEA model. Section 4 presents the efficiency measurement of the overall NDEA and sub-stages. Ultimately, to show the applicability of the proposed approach, it is applied to a real dataset in Section 5, which is followed by conclusions, and some directions for future research are given in the last section.

2 | Preliminaries

Consider the displayed structure in *Fig. 1*, which demonstrates the integration of DEA and MPI within a two-stage network system to assess the efficiency and productivity of each DMU under different scenarios. Each DMU consists of two sub-DMUs sequentially, and undesirable outputs from Stage 2 are feedbacks that can be sent back as inputs to Stage 1. Suppose there are n DMUs; in the first stage, each DMU_j $(j = 1, \dots, n)$ uses m inputs $x_{ij}^{1(t)s}(i = 1, \dots, m)$ and produces H final outputs $y_{hj}^{1(t)s}(h = 1, \dots, H)$ and D intermediate outputs $z_{dj}^{(t)s}(d = 1, \dots, D)$ based on the scenario $s \epsilon S$ that assist as the inputs to the second stage.

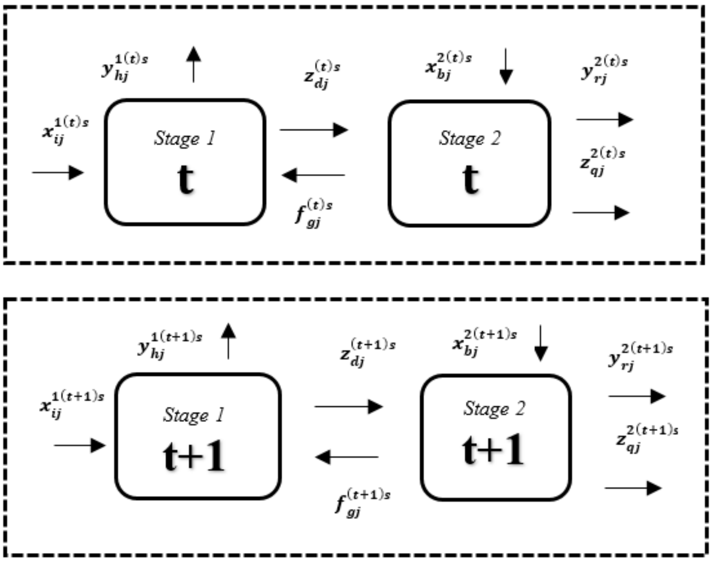


Fig. 1. A dynamic system.

Also, there are B inputs $x_{bj}^{2(t)s}(b = 1, \dots, B)$ to the second stage under scenario $s \epsilon S$. Outputs from the second stage take three forms: desirable outputs $y_{rj}^{2(t)s}(r = 1, \dots, R)$, undesirable outputs $z_{qj}^{2(t)s}(q = 1, \dots, Q)$ and a feedback variable $f_{gj}^{(t)s}(g = 1, \dots, G)$ in time t based on the scenario $s \epsilon S$. For each DMUj, the efficiency scores of the first and the second stages are denoted by $e_1^{(t)s}(t)$ and $e_2^{(t)s}(t)$, respectively, under the s^{th} scenario when all DMUs under evaluation are in time t. Also, the efficiency score of the overall process when all DMUs under assessment are in period t under the s^{th} scenario is shown by $e_o^{(t)*}(t)$.

2.1 | Undesirable Outputs

Modeling undesirable outputs and damaging side-effects of production activities have attracted considerable attention among production economists. A production process may consist of both desirable and undesirable outputs. To take care of undesirable outputs in DEA models, different approaches are developed in the DEA literature (see Chavas and Cox [37]); Hailu and Veeman [38]). Weak disposability is an alternative method that models undesirable emissions as outputs, imposing an assumption that these undesirable outputs are weakly disposable. In general, weak disposability means that it is possible to abate emissions by decreasing the level of production activity. Kuosmanen [1] defined a production technology using a weakly disposable axiom of outputs to model undesirable outputs in the DEA framework. Based on this technology, inputs, and desirable outputs are presented to be freely disposable. The weak disposability hypothesis is used to propose a modern DEA approach for evaluating the efficiency of DMUs by taking undesirable outputs into account. This approach is a significant development in computing the efficiency of DMUs with undesirable outputs. The linear programming model of this technology to evaluate the performance of a DMU in time intervals of t is as follows (Kuosmanen and Kazemi [39]):

$$\max \sum_{r=1}^{A} u_r y_{rj_o}^{2(t)s} - \sum_{q=1}^{D} \vartheta_q z_{qj_o}^{2(t)s},$$

$$\text{s.t.} \qquad (1)$$

$$\sum_{r=1}^{A} u_r y_{rj}^{2(t)s} - \sum_{q=1}^{D} \vartheta_q z_{qj}^{2(t)s} + \sum_{i=1}^{m} v_i x_{ij}^{1(t)s} \leq 0, \quad \text{for all } j, \quad \text{for all } s \epsilon S,$$

$u_r, v_i \geq 0$,   for all r, i, ϑ_q free   for all q.

In *Model (1)*, v_i, u_r, and ϑ_q are decision variables of inputs, desirable outputs, and undesirable outputs, respectively. *Constraints (2)* guarantee that the efficiency value is less than or equal to one for each DMU.

2.2 | Stochastic P-Robust Concept

Let S be a collection of scenarios, and $P^{(t)s}$ be a deterministic maximization problem for each scenario s in time t (there is a different problem $P^{(t)s}$ for each scenario $s \in S$). For each s, let $M_0^{(t)s*} > 0$ be the optimal efficiency score for $P^{(t)s}$ in time t. So, suppose that X is a feasible solution to $P^{(t)s}$ for all $s \in S$, and let $M_0^{(t)s}(X)$ be the efficiency score of $P^{(t)s}$ under solution X in time t. Then X is called p-robust if for all $s \in S$, the following inequality holds:

$$p \geq \frac{M_0^{(t)s*} - M_0^{(t)s}(X)}{M_0^{(t)s*}}. \tag{2}$$

In *Eq. (2)*, the right-hand side is the relative regret for scenarios in time t, and $p \geq 0$ is a constant that limits the relative regret for each scenario. It is obvious that inequality *Eq. (2)* can be written as below:

$$(1-p)\, M_0^{(t)s*} \leq M_0^{(t)s}(X). \tag{3}$$

Therefore, for controlling the relative regret relative to all scenarios, the p-robust *Constraints (3)* are added to the models.

Definition 1. DMU_j is stochastic p-robust efficient in different scenarios if and only if its optimal objective function is one.

3 | Efficiency of the Two-Stage Structure under Undesirable Outputs and Uncertainty

In this section, we first present a two-stage model in the presence of undesirable outputs, and then it is combined with a p-robust approach to handle the uncertainty. To evaluate the overall efficiency of the whole NDEA model in *Fig. 1* in period t, we compound the weighted average of the two stages as follows:

$$e_o^{(t)*}(t) = \max\,(\xi_1^{(t)s}\ e_1^{(t)s}(t) + \xi_2^{(t)s}\ e_2^{(t)s}(t)),$$

s.t.

$$\begin{aligned} e_1^{(t)s}(t) &= \frac{\sum_{h=1}^{H} \eta_h y_{hj}^{1(t)s} + \sum_{d=1}^{D} w_d\, z_{dj}^{(t)s}}{\sum_{i=1}^{m} v_i x_{ij}^{1(t)s} + \sum_{g=1}^{G} \partial_g f_{gj}^{(t)s}} \leq 1, \qquad \text{for all } j, \qquad \text{for all } s \in S, \\ e_2^{(t)s}(t) &= \frac{\sum_{r=1}^{s} u_r\, y_{rj}^{2(t)s} + \sum_{g=1}^{G} \partial_g f_{gj}^{(t)s} - \sum_{q=1}^{Q} \vartheta_q z_{qj}^{2(t)s}}{\sum_{d=1}^{D} w_d\, z_{dj}^{(t)s} + \sum_{t=1}^{T} \delta_b x_{bj}^{2(t)s}} \leq 1, \qquad \text{for all } j, \qquad \text{for all } s \in S, \end{aligned} \tag{4}$$

$$u_r, w_d, \partial_g, \delta_b, \eta_h, v_i, \vartheta_q \geq 0, \qquad \text{for all } r, d, g, b, h, i, q,$$

where on the basis of the radial CRS-DEA model of Charnes et al. [2], $e_1^{(t)s}$ and $e_2^{(t)s}$ are the efficiency values of the first and the second stages in time t, and $\xi_1^{(t)s}$ and $\xi_2^{(t)s}$ show the corresponding weights of stages, respectively, reflecting the importance of the two stages in the overall system ($\xi_1^{(t)s} + \xi_2^{(t)s} = 1$). We let $\xi_1^{(t)s} = \left(\sum_{i=1}^{m} v_i x_{ij_o}^{1(t)s} + \sum_{g=1}^{G} \partial_g f_{gj_o}^{(t)s}\right) / \left(\sum_{i=1}^{m} v_i x_{ij_o}^{1(t)s} + \sum_{g=1}^{G} \partial_g f_{gj_o}^{(t)s} + \sum_{d=1}^{D} w_d\, z_{dj_o}^{(t)s} + \sum_{t=1}^{T} \delta_b x_{bj_o}^{2(t)s}\right)$ and $\xi_2^{(t)s} =$

$\left(\sum_{d=1}^{D} w_d z_{dj_o}^{(t)s} + \sum_{t=1}^{T} \delta_b x_{bj_o}^{2(t)s}\right) / \left(\sum_{i=1}^{m} v_i x_{ij_o}^{1(t)s} + \sum_{g=1}^{G} \partial_g f_{gj_o}^{(t)s} + \sum_{d=1}^{D} w_d z_{dj_o}^{(t)s} + \sum_{t=1}^{T} \delta_b x_{bj_o}^{2(t)s}\right)$ in order to linearize the model. Therefore, *Model (4)* becomes:

$$e_o^{(t)s}(t) = \max \frac{\sum_{h=1}^{H} \eta_h y_{hj_o}^{1(t)s} + \sum_{d=1}^{D} w_d\, z_{dj_o}^{(t)s} + \sum_{r=1}^{s} u_r\, y_{rj_o}^{2(t)s} + \sum_{g=1}^{G} \partial_g f_{gj_o}^{(t)s} - \sum_{q=1}^{Q} \vartheta_q z_{qj_o}^{2(t)s}}{\sum_{i=1}^{m} v_i x_{ij_o}^{1(t)s} + \sum_{g=1}^{G} \partial_g f_{gj_o}^{(t)s} + \sum_{d=1}^{D} w_d\, z_{dj_o}^{(t)s} + \sum_{t=1}^{T} \delta_b x_{bj_o}^{2(t)s}},$$

s.t.

$$e_1^{(t)s}(t) = \frac{\sum_{h=1}^{H} \eta_h y_{hj}^{1(t)s} + \sum_{d=1}^{D} w_d\, z_{dj}^{(t)s}}{\sum_{i=1}^{m} v_i x_{ij}^{1(t)s} + \sum_{g=1}^{G} \partial_g f_{gj}^{(t)s}} \le 1, \quad \text{for all j,} \quad \text{for all s}\epsilon\text{S}, \tag{5}$$

$$e_2^{(t)s}(t) = \frac{\sum_{r=1}^{s} u_r\, y_{rj}^{2(t)s} + \sum_{g=1}^{G} \partial_g f_{gj}^{(t)s} - \sum_{q=1}^{Q} \vartheta_q z_{qj}^{2(t)s}}{\sum_{d=1}^{D} w_d\, z_{dj}^{(t)s} + \sum_{t=1}^{T} \delta_b x_{bj}^{2(t)s}} \le 1, \quad \text{for all j,} \quad \text{for all s}\epsilon\text{S},$$

$$u_r, w_d, \partial_g, \ \delta_b, \eta_h, \ v_i, \vartheta_q \ge 0, \quad \text{for all r, d, g, b, h.}$$

Now, let $t_1 = \left(\sum_{i=1}^{m} v_i x_{ij_o}^{1(t)s} + \sum_{g=1}^{G} \partial_g f_{gj_o}^{(t)s} + \sum_{d=1}^{D} w_d\, z_{dj_o}^{(t)s} + \sum_{t=1}^{T} \delta_b x_{bj_o}^{2(t)s}\right)^{-1}$, $\bar{v}_i = t_1 v_i$, $\bar{\partial}_g = t_1\, \partial_g$, $\bar{\eta}_h = t_1 \eta_h$, $\bar{u}_r = t_1 u_r$, $\bar{\delta}_b = t_1 \delta_b$, $\bar{w}_d = t_1 w_d$ and $\bar{\vartheta}_q = t_1 \vartheta_q$, then *Model (5)* is transformed into the following linear model:

$$e_o^{(t)s*}(t) = \max \left(\sum_{h=1}^{H} \bar{\eta}_h y_{hj_o}^{1(t)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj}^{(t)s} + \sum_{r=1}^{s} \bar{u}_r\, y_{rj_o}^{2(t)s} + \sum_{g=1}^{G} \bar{\partial}_g f_{gj_o}^{(t)s} - \sum_{q=1}^{Q} \bar{\vartheta}_q z_{qj_o}^{2(t)s}\right),$$

s.t.

$$\sum_{h=1}^{H} \bar{\eta}_h y_{hj}^{1(t)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj}^{(t)s} - \sum_{i=1}^{m} \bar{v}_i x_{ij}^{1(t)s} - \sum_{g=1}^{G} \bar{\partial}_g f_{gj}^{(t)s} \le 0, \quad \text{for all j, for all s}\epsilon\text{S},$$

$$\sum_{r=1}^{s} \bar{u}_r\, y_{rj}^{2(t)s} + \sum_{g=1}^{G} \bar{\partial}_g f_{gj}^{(t)s} - \sum_{q=1}^{Q} \bar{\vartheta}_q z_{qj}^{2(t)s} - \sum_{d=1}^{D} \bar{w}_d\, z_{dj}^{(t)s} - \sum_{t=1}^{T} \bar{\delta}_b x_{bj}^{2(t)s} \le 0, \quad \text{for all j, for all s}\epsilon\text{S}, \tag{6}$$

$$\sum_{i=1}^{m} \bar{v}_i x_{ij_o}^{1(t)s} + \sum_{g=1}^{G} \bar{\partial}_g f_{gj_o}^{(t)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj_o}^{(t)s} + \sum_{t=1}^{T} \bar{\delta}_b x_{bj_o}^{2(t)s} = 1, \quad \text{for all s}\epsilon\text{S},$$

$$u_r, w_d, \partial_g, \ \delta_b, \eta_h, \ v_i, \vartheta_q \ge 0, \quad \text{for all r, d, g, b, h.}$$

Definition 2. The two-stage process is efficient if and only if $e_1^{(t)s}(t) = e_2^{(t)s}(t) = 1$.

3.1 | Efficiency Value of the Two-Stage Structure under Uncertainty

In this section, to take care of uncertainty, *Formula (3)* can be merged with the expected objective function of *Model (6)*, and in order to control the relative regret related to the scenarios, the p-robust restrictions are added to *Model (6)*. Thus, the efficiency value for the stochastic p-robust version of *Model (6)* is as follows:

$$M_0^{(t)s*}(t) = \max \sum_{s=1}^{S} q^s \left[\sum_{h=1}^{H} \bar{\eta}_h y_{hj_o}^{1(t)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj}^{(t)s} + \sum_{r=1}^{s} \bar{u}_r\, y_{rj_o}^{2(t)s} + \sum_{g=1}^{G} \bar{\partial}_g f_{gj_o}^{(t)s} - \sum_{q=1}^{Q} \bar{\vartheta}_q z_{qj_o}^{2(t)s}\right], \tag{7.a}$$

s.t.

$$\sum_{h=1}^{H}\bar{\eta}_h y_{hj_o}^{1(t)s}+\sum_{d=1}^{D}\bar{w}_d\, z_{dj}^{(t)s}+\sum_{r=1}^{s}\bar{u}_r\, y_{rj_o}^{2(t)s} + \sum_{g=1}^{G}\bar{\partial}_g f_{gj_o}^{(t)s}-\sum_{q=1}^{Q}\bar{\vartheta}_q z_{qj_o}^{2(t)s}\geq(1-p)e_0^{(t)s*},\quad \text{for all } s\epsilon S, \tag{7.b}$$

$$\sum_{h=1}^{H}\bar{\eta}_h y_{hj}^{1(t)s}+\sum_{d=1}^{D}\bar{w}_d\, z_{dj}^{(t)s}-\sum_{i=1}^{m}\bar{v}_1 x_{ij}^{1(t)s}-\sum_{g=1}^{G}\bar{\partial}_g f_{gj}^{(t)s}\leq 0,\quad \text{for all j, for all } s\epsilon S, \tag{7.c}$$

$$\sum_{r=1}^{s}\bar{u}_r\, y_{rj}^{2(t)s}+\sum_{g=1}^{G}\bar{\partial}_g f_{gj}^{(t)s}-\sum_{q=1}^{Q}\bar{\vartheta}_q z_{qj}^{2(t)s}-\sum_{d=1}^{D}\bar{w}_d\, z_{dj}^{(t)s}-\sum_{t=1}^{T}\bar{\delta}_b x_{bj}^{2(t)s} \leq 0,\quad \text{for all j, for all } s\epsilon S, \tag{7.d}$$

$$\sum_{i=1}^{m}\bar{v}_1 x_{ij_o}^{1(t)s}+\sum_{g=1}^{G}\bar{\partial}_g f_{gj_o}^{(t)s}+\sum_{d=1}^{D}\bar{w}_d\, z_{dj_o}^{(t)s}+\sum_{t=1}^{T}\bar{\delta}_b x_{bj_o}^{2(t)s}=1,\quad \text{for all } s\epsilon S, \tag{7.e}$$

$$u_r, w_d, \partial_g, \delta_b, \eta_h, v_i, \vartheta_q \geq 0,\quad \text{for all r, d, g, b, h.} \tag{7.f}$$

It is noteworthy that models for the first and the second stages can be linearized, similar to the overall efficiency. *Model (7)* evaluates the relative efficiency of the whole system, and the data for all DMUs are retrieved from period t. The objective function of *Model (7)* is to maximize the expected efficiency value of all DMUs. Further, q^s in the objective function is the probability that scenario s happens. The first constraint in all models is called the p-robust constraint, which may not allow the scenario efficiency to take a value of more than $100(1-p)\%$ of the ideal efficiency scores gained by each scenario. The parameter p controls the relative regret between all scenarios. The p-robust constraints in this model become ineffective if $p=\infty$. It is noteworthy that the p-values generally are assumed to be greater than 0.2, and their upper bound is gained by try and error. Also, these values can be different for any problem and are usually defined by the decision-maker.

Definition 3. If $M_0^{(t)s*}(t)=1$ in *Model (7)*, then DMU_o is efficient.

4 | The MPI Models in the Presence of Undesirable Output and Uncertainty

In this section, we compute the efficiency of the overall NDEA process in periods t and $t+1$. To measure the efficiency of *Model (7)* when the data for the DMU under evaluation is retaken from period $t+1$ while the data for the other DMUs are retaken from period t, the following model applies:

$$M_0^{(t+1)s*}(t)=\max\sum_{s=1}^{S}q^s\left[\sum_{h=1}^{H}\bar{\eta}_h y_{hj_o}^{1(t+1)s}+\sum_{d=1}^{D}\bar{w}_d\, z_{dj}^{(t+1)s}+\sum_{r=1}^{s}\bar{u}_r\, y_{rj_o}^{2(t+1)s}+ \sum_{g=1}^{G}\bar{\partial}_g f_{gj_o}^{(t+1)s}-\sum_{q=1}^{Q}\bar{\vartheta}_q z_{qj_o}^{2(t+1)s}\right], \tag{8.a}$$

s.t.

$$\sum_{h=1}^{H}\bar{\eta}_h y_{hj_o}^{1(t+1)s}+\sum_{d=1}^{D}\bar{w}_d\, z_{dj_o}^{(t+1)s}+\sum_{r=1}^{s}\bar{u}_r\, y_{rj_o}^{2(t+1)s}+\sum_{g=1}^{G}\bar{\partial}_g f_{gj_o}^{(t+1)s}-\sum_{q=1}^{Q}\bar{\vartheta}_q z_{qj_o}^{2(t+1)s}\geq(1-p)e_0^{(t+1)s*},\ \text{for all } s\epsilon S, \tag{8.b}$$

$$\sum_{h=1}^{H}\bar{\eta}_h y_{hj}^{1(t)s}+\sum_{d=1}^{D}\bar{w}_d\, z_{dj}^{(t)s}-\sum_{i=1}^{m}\bar{v}_i x_{ij}^{1(t)s}-\sum_{g=1}^{G}\bar{\partial}_g f_{gj}^{(t)s}\leq 0,\quad \text{for all } j, \text{for all } s\epsilon S, \tag{8.c}$$

$$\sum_{h=1}^{H}\bar{\eta}_h y_{hj_o}^{1(t+1)s}+\sum_{d=1}^{D}\bar{w}_d\, z_{dj_o}^{(t+1)s}-\sum_{i=1}^{m}\bar{v}_i x_{ij_o}^{1(t+1)s}-\sum_{g=1}^{G}\bar{\partial}_g f_{gj_o}^{(t+1)s}\leq 0,\quad \text{for all } j,\ \text{for all } s\epsilon S, \tag{8.d}$$

$$\sum_{r=1}^{s}\bar{u}_r\, y_{rj}^{2(t)s}+\sum_{g=1}^{G}\bar{\partial}_g f_{gj}^{(t)s}-\sum_{q=1}^{Q}\bar{\vartheta}_q z_{qj}^{2(t)s}-\sum_{d=1}^{D}\bar{w}_d\, z_{dj}^{(t)s}-\sum_{t=1}^{T}\bar{\delta}_b x_{bj}^{2(t)s}\leq 0,\quad \text{for all } j, \text{for all } s\epsilon S, \tag{8.e}$$

$$\sum_{r=1}^{s}\bar{u}_r\, y_{rj_o}^{2(t+1)s}+\sum_{g=1}^{G}\bar{\partial}_g f_{gj_o}^{(t+1)s}-\sum_{q=1}^{Q}\bar{\vartheta}_q z_{qj_o}^{2(t+1)s}-\sum_{d=1}^{D}\bar{w}_d\, z_{dj_o}^{(t+1)s}-\sum_{t=1}^{T}\bar{\delta}_b x_{bj_o}^{2(t+1)s}\leq 0,\quad \text{for all } j,\ \text{for all } s\epsilon S, \tag{8.f}$$

$$\sum_{i=1}^{m}\bar{v}_i x_{ij_o}^{1(t+1)s}+\sum_{g=1}^{G}\bar{\partial}_g f_{gj_o}^{(t+1)s}+\sum_{d=1}^{D}\bar{w}_d\, z_{dj_o}^{(t+1)s}+\sum_{t=1}^{T}\bar{\delta}_b x_{bj_o}^{2(t+1)s}=1,\quad \text{for all } s\epsilon S,$$
$$u_r, w_d, \partial_g, \delta_b, \eta_h, v_i, \vartheta_q\geq 0,\qquad \text{for all } r, d, g, b, h \tag{8.g}$$

Similarly, to scale the efficiency of the overall network when the data for the DMU under the estimate are recovered from period t while the data for the other DMUs are recovered from period $t+1$ *Model (9)* is provided:

$$M_0^{(t)s*}(t+1)=\max\sum_{s=1}^{S}q^s\left[\sum_{h=1}^{H}\bar{\eta}_h y_{hj_o}^{1(t)s}+\sum_{d=1}^{D}\bar{w}_d\, z_{dj}^{(t)s}+\sum_{r=1}^{s}\bar{u}_r\, y_{rj_o}^{2(t)s}+\sum_{g=1}^{G}\bar{\partial}_g f_{gj_o}^{(t)s}-\sum_{q=1}^{Q}\bar{\vartheta}_q z_{qj_o}^{2(t)s}\right], \tag{9.a}$$

s.t.

$$\sum_{h=1}^{H}\bar{\eta}_h y_{hj_o}^{1(t)s}+\sum_{d=1}^{D}\bar{w}_d\, z_{dj_o}^{(t)s}+\sum_{r=1}^{s}\bar{u}_r\, y_{rj_o}^{2(t)s}+\sum_{g=1}^{G}\bar{\partial}_g f_{gj_o}^{(t)s}-\sum_{q=1}^{Q}\bar{\vartheta}_q z_{qj_o}^{2(t)s}\geq(1-p)e_0^{(t)s*},\ \text{for all } s\epsilon S, \tag{9.b}$$

$$\sum_{h=1}^{H}\bar{\eta}_h y_{hj}^{1(t+1)s}+\sum_{d=1}^{D}\bar{w}_d\, z_{dj}^{(t+1)s}-\sum_{i=1}^{m}\bar{v}_i x_{ij}^{1(t+1)s}-\sum_{g=1}^{G}\bar{\partial}_g f_{gj}^{(t+1)s}\leq 0,\quad \text{for all } j, \text{for all } s\epsilon S, \tag{9.c}$$

$$\sum_{h=1}^{H}\bar{\eta}_h y_{hj_o}^{1(t)s}+\sum_{d=1}^{D}\bar{w}_d\, z_{dj_o}^{(t)s}-\sum_{i=1}^{m}\bar{v}_i x_{ij_o}^{1(t)s}-\sum_{g=1}^{G}\bar{\partial}_g f_{gj_o}^{(t)s}\leq 0,\quad \text{for all } j, \text{for all } s\epsilon S, \tag{9.d}$$

$$\sum_{r=1}^{s} \bar{u}_r\, y_{rj}^{2(t+1)s} + \sum_{g=1}^{G} \bar{\partial}_g f_{gj}^{(t+1)s} - \sum_{q=1}^{Q} \bar{\vartheta}_q z_{qj}^{2(t+1)s} - \sum_{d=1}^{D} \bar{w}_d\, z_{dj}^{(t+1)s} - \sum_{t=1}^{T} \bar{\delta}_b x_{bj}^{2(t+1)s} \leq 0, \quad \text{for all } j, \quad \text{for all } s\epsilon S, \tag{9.e}$$

$$\sum_{r=1}^{s} \bar{u}_r\, y_{rj_o}^{2(t)s} + \sum_{g=1}^{G} \bar{\partial}_g f_{gj_o}^{(t)s} - \sum_{q=1}^{Q} \bar{\vartheta}_q z_{qj_o}^{2(t)s} - \sum_{d=1}^{D} \bar{w}_d\, z_{dj_o}^{(t)s} - \sum_{t=1}^{T} \bar{\delta}_b x_{bj_o}^{2(t)s} \leq 0, \quad \text{for all } j, \quad \text{for all } s\epsilon S, \tag{9.f}$$

$$\sum_{i=1}^{m} \bar{v}_l x_{ij_o}^{1(t)s} + \sum_{g=1}^{G} \bar{\partial}_g f_{gj_o}^{(t)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj_o}^{(t)s} + \sum_{t=1}^{T} \bar{\delta}_b x_{bj_o}^{2(t)s} = 1, \quad \text{for all } s\epsilon S,$$

$$u_r,\ w_d,\ \partial_g,\ \delta_b,\ \eta_h,\ v_i,\ \vartheta_q \geq 0, \quad \text{for all } r, d, g, t, h. \tag{9.g}$$

Eventually, *Model (10)* is suggested to compute the relative efficiency of the overall NDEA when the data for all DMUs, containing the DMU underestimate, are retaken from period $t+1$ based on scenario s as below:

$$M_0^{(t+1)s*}(t+1) = \max \sum_{s=1}^{S} q^s \Big[\sum_{h=1}^{H} \bar{\eta}_h y_{hj_o}^{1(t+1)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj}^{(t+1)s} + \sum_{r=1}^{s} \bar{u}_r\, y_{rj_o}^{2(t+1)s} + \sum_{g=1}^{G} \bar{\partial}_g f_{gj_o}^{(t+1)s} - \sum_{q=1}^{Q} \bar{\vartheta}_q z_{qj_o}^{2(t+1)s}\Big], \tag{10.a}$$

s.t

$$\sum_{h=1}^{H} \bar{\eta}_h y_{hj_o}^{1(t+1)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj}^{(t+1)s} + \sum_{r=1}^{s} \bar{u}_r\, y_{rj_o}^{2(t+1)s} + \sum_{g=1}^{G} \bar{\partial}_g f_{gj_o}^{(t+1)s} - \sum_{q=1}^{Q} \bar{\vartheta}_q z_{qj_o}^{2(t+1)s} \geq (1-p) e_0^{(t+1)s*}, \quad \text{for all } s\epsilon S, \tag{10.b}$$

$$\sum_{h=1}^{H} \bar{\eta}_h y_{hj}^{1(t+1)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj}^{(t+1)s} - \sum_{i=1}^{m} \bar{v}_l x_{ij}^{1(t+1)s} - \sum_{g=1}^{G} \bar{\partial}_g f_{gj}^{(t+1)s} \leq 0, \quad \text{for all } j, \quad \text{for all } s\epsilon S, \tag{10.c}$$

$$\sum_{r=1}^{s} \bar{u}_r\, y_{rj}^{2(t+1)s} + \sum_{g=1}^{G} \bar{\partial}_g f_{gj}^{(t+1)s} - \sum_{q=1}^{Q} \bar{\vartheta}_q z_{qj}^{2(t+1)s} - \sum_{d=1}^{D} \bar{w}_d\, z_{dj}^{(t+1)s} - \sum_{t=1}^{T} \bar{\delta}_b x_{bj}^{2(t+1)s} \leq 0, \quad \text{for all } j, \quad \text{for all } s\epsilon S, \tag{10.d}$$

$$\sum_{i=1}^{m} \bar{v}_l x_{ij_o}^{1(t+1)s} + \sum_{g=1}^{G} \bar{\partial}_g f_{gj_o}^{(t+1)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj_o}^{(t+1)s} + \sum_{t=1}^{T} \bar{\delta}_b x_{bj_o}^{2(t+1)s} = 1, \quad \text{for all } s\epsilon S, \tag{10.e}$$

$$u_r, w_d, \partial_g,\ \delta_b, \eta_h,\ v_i, \vartheta_q \geq 0, \quad \text{for all } r, d, g, b, h. \tag{10.f}$$

4.1 | Efficiency Scaling of the First Stage Through Both Periods

In this section, the efficiency of the first stage in the presence of undesirable outputs and uncertainty is calculated. *Model (11)* computes the maximum achievable value for the efficiency of the first stage under the s^{th} scenario in periods t and $t+1$:

$$M_{10}^{(t)s*}(t) = \max \sum_{s=1}^{S} q^s \left[\sum_{h=1}^{H} \bar{\eta}_h y_{hj_o}^{1(t)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj_o}^{(t)s}\right], \tag{11.a}$$

s.t.

$$\sum_{h=1}^{H} \bar{\eta}_h y_{hj_o}^{1(t)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj_o}^{(t)s} \geq (1-p) e_{10}^{(t)s*}, \qquad \text{for all } s \epsilon S, \tag{11.b}$$

$$\sum_{h=1}^{H} \bar{\eta}_h y_{hj}^{1(t)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj}^{(t)s} - \sum_{i=1}^{m} \bar{v}_i x_{ij}^{1(t)s} - \sum_{g=1}^{G} \bar{\partial}_g f_{gj}^{(t)s} \leq 0, \ \text{for all } j, \qquad \text{for all } s \epsilon S, \tag{11.c}$$

$$\sum_{i=1}^{m} \bar{v}_i x_{ij_o}^{1(t)s} + \sum_{g=1}^{G} \bar{\partial}_g f_{gj_o}^{(t)s} = 1, \qquad \text{for all } s \epsilon S,$$
$$\bar{\partial}_g, \bar{w}_d, \bar{\eta}_h, \bar{v}_i \geq 0, \qquad \text{for all } g, d, h, i. \tag{11.d}$$

Remark 1. By considering $f_{ko}^{(t)s} = \max\left\{f_{go}^{(t)s} \middle| 1 \leq g \leq G\right\} > 0$, $x_{ko}^{(t)s} = \max\left\{x_{io}^{1(t)s} \middle| 1 \leq i \leq m\right\} > 0$ and then setting $(\bar{\eta}_1, \dots, \bar{w}_1, \dots, , \bar{v}_1, \dots, \bar{\partial}_1, \dots) = \left(0, \dots, 1/x_{ko}^{1(t)s}, 0, \dots, 1/f_{ko}^{(t)s}, 0, \dots\right)$, restrictions $\sum_{i=1}^{m} \bar{v}_i x_{ij_o}^{1(t)s} + \sum_{g=1}^{G} \bar{\partial}_g f_{gj_o}^{(t)s} = 1$, **(11.e)** and $\sum_{h=1}^{H} \bar{\eta}_h y_{hj}^{1(t)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj}^{(t)s} - \sum_{i=1}^{m} \bar{v}_i x_{ij}^{1(t)s} - \sum_{g=1}^{G} \bar{\partial}_g f_{gj}^{(t)s} \leq 0$, imply $\sum_{h=1}^{H} \bar{\eta}_h y_{hj_o}^{1(t)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj_o}^{(t)s} \leq 1$. Thus, we achieve $e_{10}^{(t)s*} \leq \frac{1}{1-p}$.

So, for very small p's, there may not be p-robust solutions for *Model (11)* in periods t and $t+1$ based on scenario s; therefore, it may be infeasible.*Model (12)* is presented to scale the efficiency of Stage 1 when the data for the DMU in evaluation are recovered from period $t+1$ while the data for the other DMUs are recovered from period t as below:

$$M_{10}^{(t+1)s*}(t) = \max \sum_{s=1}^{S} q^s \left[\sum_{h=1}^{H} \bar{\eta}_h y_{hj_o}^{1(t+1)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj_o}^{(t+1)s}\right], \tag{12.a}$$

s.t.

$$\sum_{h=1}^{H} \bar{\eta}_h y_{hj_o}^{1(t+1)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj_o}^{(t+1)s} \geq (1-p) e_{10}^{(t+1)s*}, \quad \text{for all } s \epsilon S, \tag{12.b}$$

$$\sum_{h=1}^{H} \bar{\eta}_h y_{hj}^{1(t)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj}^{(t)s} - \sum_{i=1}^{m} \bar{v}_i x_{ij}^{1(t)s} - \sum_{g=1}^{G} \bar{\partial}_g f_{gj}^{(t)s} \leq 0, \ \text{for all } j, \qquad \text{for all } s \epsilon S, \tag{12.c}$$

$$\sum_{h=1}^{H} \bar{\eta}_h y_{hj_o}^{1(t+1)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj_o}^{(t+1)s} - \sum_{i=1}^{m} \bar{v}_i x_{ij_o}^{1(t+1)s} - \sum_{g=1}^{G} \bar{\partial}_g f_{gj_o}^{(t+1)s} \leq 0, \ \text{for all } j, \quad \text{for all } s \epsilon S, \tag{12.d}$$

$$\sum_{i=1}^{m} \bar{v}_i x_{ij_o}^{1(t+1)s} + \sum_{g=1}^{G} \bar{\partial}_g f_{gj_o}^{(t+1)s} = 1, \qquad \text{for all } s \epsilon S,$$
$$\bar{\partial}_g, \bar{w}_d, \bar{\eta}_h, \bar{v}_i \geq 0, \qquad \text{for all } g, d, h, i. \tag{12.f}$$

Also, *Model (13)* is applied to scale the efficiency of Stage 1 when the data for the DMU in evaluation are recovered from period t while the data for the other DMUs are recovered from period $t+1$:

$$M_{10}^{(t)s*}(t+1) = \max \sum_{s=1}^{S} q^s \left[\sum_{h=1}^{H} \bar{\eta}_h y_{hj_o}^{1ts} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj_o}^{ts}\right], \tag{13.a}$$

s.t.

$$\sum_{h=1}^{H} \bar{\eta}_h y_{hj_o}^{1ts} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj_o}^{ts} \geq (1-p) e_{10}^{ts*}, \quad \text{for all } s \epsilon S, \tag{13.b}$$

$$\sum_{h=1}^{H} \bar{\eta}_h y_{hj}^{1(t+1)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj}^{(t+1)s} - \sum_{i=1}^{m} \bar{v}_i x_{ij}^{1(t+1)s} - \sum_{g=1}^{G} \bar{\partial}_g f_{gj}^{(t+1)s} \leq 0, \quad \text{for all j, for all } s\epsilon S, \tag{13.c}$$

$$\sum_{i=1}^{m} \bar{v}_i x_{ij_o}^{1ts} + \sum_{g=1}^{G} \bar{\partial}_g f_{gj_o}^{ts} = 1, \quad \text{for all } s\epsilon S, \tag{13.d}$$

$$\bar{\partial}_g, \bar{w}_d, \bar{\eta}_h, \bar{v}_i \geq 0, \quad \text{for all g, d, h, i.} \tag{13.e}$$

Finally, *Model (14)* is presented to evaluate the efficiency of Stage 1 when the data for the DMU under evaluation are retaken from period t while the data for the other DMUs are retaken from period $t + 1$:

$$M_{10}^{(t+1)s*}(t+1) = \max \sum_{s=1}^{S} q^s \left[\sum_{h=1}^{H} \bar{\eta}_h y_{hj_o}^{1(t+1)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj_o}^{(t+1)s} \right], \tag{14.a}$$

s.t.

$$\sum_{h=1}^{H} \bar{\eta}_h y_{hj_o}^{1(t+1)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj_o}^{(t+1)s} \geq (1-p) e_{10}^{(t+1)s*}, \quad \text{for all } s\epsilon S, \tag{14.b}$$

$$\sum_{h=1}^{H} \bar{\eta}_h y_{hj}^{1(t+1)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj}^{(t+1)s} - \sum_{i=1}^{m} \bar{v}_i x_{ij}^{1(t+1)s} - \sum_{g=1}^{G} \bar{\partial}_g f_{gj}^{(t+1)s} \leq 0, \quad \text{for all j, for all } s\epsilon S, \tag{14.c}$$

$$\sum_{i=1}^{m} \bar{v}_i x_{ij_o}^{1(t+1)s} + \sum_{g=1}^{G} \bar{\partial}_g f_{gj_o}^{(t+1)s} = 1, \quad \text{for all } s\epsilon S, \tag{14.d}$$

$$\bar{\partial}_g, \bar{w}_d, \bar{\eta}_h, \bar{v}_i \geq 0, \quad \text{for all g, d, h, i.} \tag{14.e}$$

4.2 | Efficiency Assessment of the Second Stage Through Both Periods

The efficiency value of the second stage in the presence of undesirable outputs and uncertainty is defined as *Model (15)*. It evaluates the efficiency of Stage 2 when the data for the DMU undervaluation are regained from period $t + 1$ while the data for the other DMUs are regained from period t:

$$M_{20}^{(t)s*}(t) = \max \sum_{s=1}^{S} q^s \left[\sum_{r=1}^{s} \bar{u}_r\, y_{rj_o}^{2(t)s} + \sum_{g=1}^{G} \bar{\partial}_g f_{gj_o}^{(t)s} - \sum_{q=1}^{Q} \bar{\vartheta}_q z_{qj_o}^{2(t)s} \right], \tag{15.a}$$

s.t.

$$\sum_{r=1}^{s} \bar{u}_r\, y_{rj_o}^{2(t)s} + \sum_{g=1}^{G} \bar{\partial}_g f_{gj_o}^{(t)s} - \sum_{q=1}^{Q} \bar{\vartheta}_q z_{qj_o}^{2(t)s} \geq (1-p) e_0^{2(t)s*}, \quad \text{for all } s\epsilon S, \tag{15.b}$$

$$\sum_{r=1}^{s} \bar{u}_r\, y_{rj}^{2(t)s} + \sum_{g=1}^{G} \bar{\partial}_g f_{gj}^{(t)s} - \sum_{q=1}^{Q} \bar{\vartheta}_q z_{qj}^{2(t)s} - \sum_{d=1}^{D} \bar{w}_d\, z_{dj}^{(t)s} - \sum_{t=1}^{T} \bar{\delta}_b x_{bj}^{2(t)s} \leq 0, \quad \text{for all j, for all } s\epsilon S, \tag{15.c}$$

$$\sum_{d=1}^{D} \bar{w}_d\, z_{dj_o}^{(t)s} + \sum_{t=1}^{T} \bar{\delta}_b x_{bj_o}^{2(t)s} = 1, \quad \text{for all } s\epsilon S, \tag{15.d}$$

$$\sum_{h=1}^{H} \bar{\eta}_h y_{hj}^{1(t)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj}^{(t)s} - \sum_{i=1}^{m} \bar{v}_i x_{ij}^{1(t)s} - \sum_{g=1}^{G} \bar{\partial}_g f_{gj}^{(t)s} \leq 0, \quad \text{for all j, for all } s\epsilon S, \tag{15.e}$$

$$\sum_{h=1}^{H} \bar{\eta}_h y_{hj_o}^{1(t)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj_o}^{(t)s} - e_0^{1(t)s*}\left(\sum_{i=1}^{m} \bar{v}_i x_{ij_o}^{1(t)s} + \sum_{g=1}^{G} \bar{\partial}_g f_{gj_o}^{(t)s}\right) = 0, \quad \text{for all } s\epsilon S, \tag{15.f}$$

$$u_r, \bar{w}_d, \partial_g, \delta_b, \eta_h, v_i, \vartheta_q > 0, \quad \text{for all r, d, g, b, h, i, q.} \tag{15.g}$$

Model (16) is expanded to measure the efficiency value of Stage 2 as follows, where DMU at period $t+1$ and the frontier at period t:

$$M_{20}^{(t+1)s*}(t) = \max \sum_{s=1}^{S} q^s \left[\sum_{r=1}^{s} \bar{u}_r\, y_{rj_o}^{2(t)s} + \sum_{g=1}^{G} \bar{\partial}_g f_{gj_o}^{(t)s} - \sum_{q=1}^{Q} \bar{\vartheta}_q z_{qj_o}^{2(t)s}\right], \tag{16.a}$$

s.t.

$$\sum_{r=1}^{s} \bar{u}_r\, y_{rj_o}^{2(t+1)s} + \sum_{g=1}^{G} \bar{\partial}_g f_{gj_o}^{(t+1)s} - \sum_{q=1}^{Q} \bar{\vartheta}_q z_{qj_o}^{2(t+1)s} \geq (1-p) e_0^{2(t+1)s*}, \quad \text{for all } s\epsilon S, \tag{16.b}$$

$$\sum_{r=1}^{s} \bar{u}_r\, y_{rj}^{2(t)s} + \sum_{g=1}^{G} \bar{\partial}_g f_{gj}^{(t)s} - \sum_{q=1}^{Q} \bar{\vartheta}_q z_{qj}^{2(t)s} - \sum_{d=1}^{D} \bar{w}_d\, z_{dj}^{(t)s} - \sum_{t=1}^{T} \bar{\delta}_b x_{bj}^{2(t)s} \leq 0, \text{ for all j, for all } s\epsilon S, \tag{16.c}$$

$$\sum_{r=1}^{s} \bar{u}_r\, y_{rj_o}^{2(t+1)s} + \sum_{g=1}^{G} \bar{\partial}_g f_{gj_o}^{(t+1)s} - \sum_{q=1}^{Q} \bar{\vartheta}_q z_{qj_o}^{2(t+1)s} - \sum_{d=1}^{D} \bar{w}_d\, z_{dj_o}^{(t+1)s} - \sum_{t=1}^{T} \bar{\delta}_b x_{bj_o}^{2(t+1)s} \leq 0, \text{ for all j, for all } s\epsilon S, \tag{16.d}$$

$$\sum_{d=1}^{D} \bar{w}_d\, z_{dj_o}^{(t+1)s} + \sum_{t=1}^{T} \bar{\delta}_b x_{bj_o}^{2(t+1)s} = 1, \quad \text{for all } s\epsilon S, \tag{16.e}$$

$$\sum_{h=1}^{H} \bar{\eta}_h y_{hj}^{1(t)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj}^{(t)s} - \sum_{i=1}^{m} \bar{v}_i x_{ij}^{1(t)s} - \sum_{g=1}^{G} \bar{\partial}_g f_{gj}^{(t)s} \leq 0, \quad \text{for all j,} \quad \text{for all } s\epsilon S, \tag{16.f}$$

$$\sum_{h=1}^{H} \bar{\eta}_h y_{hj_o}^{1(t+1)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj_o}^{(t+1)s} - \sum_{i=1}^{m} \bar{v}_i x_{ij_o}^{1(t+1)s} - \sum_{g=1}^{G} \bar{\partial}_g f_{gj_o}^{(t+1)s} \leq 0, \quad \text{for all j, for all } s\epsilon S, \tag{16.g}$$

$$\sum_{h=1}^{H} \bar{\eta}_h y_{hj_o}^{1(t+1)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj_o}^{(t+1)s} - e_0^{1(t+1)s*}\left(\sum_{i=1}^{m} \bar{v}_i x_{ij_o}^{1(t+1)s} + \sum_{g=1}^{G} \bar{\partial}_g f_{gj_o}^{(t+1)s}\right) = 0, \quad \text{for all } s\epsilon S, \tag{16.h}$$

$$u_r, \bar{w}_d, \partial_g, \delta_b, \eta_h, v_i, \vartheta_q > 0, \quad \text{for all r, d, g, b, h, i, q.} \tag{16.i}$$

Likewise, *Model (16)* is introduced to compute the efficiency value of Stage 2 when the data for the DMU under evaluation are retaken from period t while the data for the other DMUs are retaken from period $t+1$ as follows:

$$M_{20}^{(t)s*}(t+1) = \max \sum_{s=1}^{S} q^s \left[\sum_{r=1}^{s} \bar{u}_r\, y_{rj_o}^{2(t)s} + \sum_{g=1}^{G} \bar{\partial}_g f_{gj_o}^{(t)s} - \sum_{q=1}^{Q} \bar{\vartheta}_q z_{qj_o}^{2(t)s}\right], \tag{17.a}$$

s.t.

$$\sum_{r=1}^{s}\bar{u}_r\, y_{rj_o}^{2(t)s}+\sum_{g=1}^{G}\bar{\partial}_g f_{gj_o}^{(t)s}-\sum_{q=1}^{Q}\bar{\vartheta}_q z_{qj_o}^{2(t)s}\geq(1-p)e_0^{2(t)s*},\quad \text{for all } s\epsilon S, \tag{17.b}$$

$$\sum_{r=1}^{s}\bar{u}_r\, y_{rj}^{2(t+1)s}+\sum_{g=1}^{G}\bar{\partial}_g f_{gj}^{(t+1)s}-\sum_{q=1}^{Q}\bar{\vartheta}_q z_{qj}^{2(t+1)s}-\sum_{d=1}^{D}\bar{w}_d\, z_{dj}^{(t+1)s}-\sum_{t=1}^{T}\bar{\delta}_b x_{bj}^{2(t+1)s} \leq 0, \text{for all } j,\quad \text{for all } s\epsilon S, \tag{17.c}$$

$$\sum_{r=1}^{s}\bar{u}_r\, y_{rj_o}^{2(t)s}+\sum_{g=1}^{G}\bar{\partial}_g f_{gj_o}^{(t)s}-\sum_{q=1}^{Q}\bar{\vartheta}_q z_{qj_o}^{2(t)s}-\sum_{d=1}^{D}\bar{w}_d\, z_{dj_o}^{(t)s}-\sum_{t=1}^{T}\bar{\delta}_b x_{bj_o}^{2(t)s} \leq 0, \text{for all } j,\quad \text{for all } s\epsilon S, \tag{17.d}$$

$$\sum_{d=1}^{D}\bar{w}_d\, z_{dj_o}^{(t)s}+\sum_{t=1}^{T}\bar{\delta}_b x_{bj_o}^{2(t)s}=1,\quad \text{for all } s\epsilon S, \tag{17.e}$$

$$\sum_{h=1}^{H}\bar{\eta}_h y_{hj}^{1(t+1)s}+\sum_{d=1}^{D}\bar{w}_d\, z_{dj}^{(t+1)s}-\sum_{i=1}^{m}\bar{v}_i x_{ij}^{1(t+1)s}-\sum_{g=1}^{G}\bar{\partial}_g f_{gj}^{(t+1)s}\leq 0,\quad \text{for all } j, \text{ for all } s\epsilon S, \tag{17.f}$$

$$\sum_{h=1}^{H}\bar{\eta}_h y_{hj_o}^{1(t)s}+\sum_{d=1}^{D}\bar{w}_d\, z_{dj_o}^{(t)s}-\sum_{i=1}^{m}\bar{v}_i x_{ij_o}^{1(t)s}-\sum_{g=1}^{G}\bar{\partial}_g f_{gj_o}^{(t)s}\leq 0,\quad \text{for all } j,\quad \text{for all } s\epsilon S, \tag{17.g}$$

$$\sum_{h=1}^{H}\bar{\eta}_h y_{hj_o}^{1(t)s}+\sum_{d=1}^{D}\bar{w}_d\, z_{dj_o}^{(t)s}-\ e_0^{1(t)s*}\left(\sum_{i=1}^{m}\bar{v}_i x_{ij_o}^{1(t)s}+\sum_{g=1}^{G}\bar{\partial}_g f_{gj_o}^{(t)s}\right)=0,\quad \text{for all } s\epsilon S, \tag{17.h}$$

$$u_r, \bar{w}_d, \partial_g,\ \delta_b, \eta_h,\ v_i, \vartheta_q>0,\quad \text{for all } r, d, g, b, h, i, q. \tag{17.i}$$

Finally, *Model (18)* computes the efficiency value of Stage 2 when the data for all DMUs containing the DMU under evaluation are retaken from period $t+1$ as follows:

$$M_{20}^{(t+1)s*}(t+1)=\max\sum_{s=1}^{S}q^s\left[\sum_{r=1}^{s}\bar{u}_r\, y_{rj_o}^{2(t+1)s}+\sum_{g=1}^{G}\bar{\partial}_g f_{gj_o}^{(t+1)s}-\sum_{q=1}^{Q}\bar{\vartheta}_q z_{qj_o}^{2(t+1)s}\right], \tag{18.a}$$

s.t.

$$\sum_{r=1}^{s}\bar{u}_r\, y_{rj_o}^{2(t+1)s}+\sum_{g=1}^{G}\bar{\partial}_g f_{gj_o}^{(t+1)s}-\sum_{q=1}^{Q}\bar{\vartheta}_q z_{qj_o}^{2(t+1)s}\geq(1-p)e_0^{2(t+1)s*},\quad \text{for all } s\epsilon S, \tag{18.b}$$

$$\sum_{r=1}^{s}\bar{u}_r\, y_{rj}^{2(t+1)s}+\sum_{g=1}^{G}\bar{\partial}_g f_{gj}^{(t+1)s}-\sum_{q=1}^{Q}\bar{\vartheta}_q z_{qj}^{2(t+1)s}-\sum_{d=1}^{D}\bar{w}_d\, z_{dj}^{(t+1)s}-\sum_{t=1}^{T}\bar{\delta}_b x_{bj}^{2(t+1)s} \leq 0,\quad \text{for all } j,\quad \text{for all } s\epsilon S, \tag{18.c}$$

$$\sum_{r=1}^{s}\bar{u}_r\, y_{rj}^{2(t+1)s}+\sum_{g=1}^{G}\bar{\partial}_g f_{gj}^{(t+1)s}-\sum_{q=1}^{Q}\bar{\vartheta}_q z_{qj}^{2(t+1)s}-\sum_{d=1}^{D}\bar{w}_d\, z_{dj}^{(t+1)s}-\sum_{t=1}^{T}\bar{\delta}_b x_{bj}^{2(t+1)s} \leq 0,\quad \text{for all } j,\quad \text{for all } s\epsilon S, \tag{18.d}$$

$$\sum_{r=1}^{s}\bar{u}_r\, y_{rj}^{2(t+1)s}+\sum_{g=1}^{G}\bar{\partial}_g f_{gj}^{(t+1)s}-\sum_{q=1}^{Q}\bar{\vartheta}_q z_{qj}^{2(t+1)s}-\sum_{d=1}^{D}\bar{w}_d\, z_{dj}^{(t+1)s}-\sum_{t=1}^{T}\bar{\delta}_b x_{bj}^{2(t+1)s} \leq 0,\quad \text{for all } j,\quad \text{for all } s\epsilon S, \tag{18.e}$$

$$\sum_{d=1}^{D}\bar{w}_d\, z_{dj_o}^{(t+1)s}+\sum_{t=1}^{T}\bar{\delta}_b x_{bj_o}^{2(t+1)s}=1,\quad \text{for all } s\epsilon S, \tag{18.f}$$

$$\sum_{h=1}^{H} \bar{\eta}_h y_{hj}^{1(t+1)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj}^{(t+1)s} - \sum_{i=1}^{m} \bar{v}_i x_{ij}^{1(t+1)s} - \sum\nolimits_{g=1}^{G} \bar{\partial}_g f_{gj}^{(t+1)s} \leq 0, \quad \text{for all j}, \quad \text{for all } s \epsilon S, \tag{18.g}$$

$$\sum_{h=1}^{H} \bar{\eta}_h y_{hj_o}^{1(t+1)s} + \sum_{d=1}^{D} \bar{w}_d\, z_{dj_o}^{(t+1)s} - e_0^{1(t+1)s*}\left(\sum_{i=1}^{m} \bar{v}_i x_{ij_o}^{1(t+1)s} + \sum\nolimits_{g=1}^{G} \bar{\partial}_g f_{gj_o}^{(t+1)s}\right) = 0, \quad \text{for all } s \epsilon S, \tag{18.h}$$

$$u_r, \bar{w}_d, \partial_g, \delta_b, \eta_h, v_i, \vartheta_q > 0, \quad \text{for all r, d, g, b, h, i, q.} \tag{18.i}$$

Finally, to assess the progress or regress of a DMU, the MPI is calculated using *Eq. (19)* as technological references to scale the variation in productivity taking place across periods t and t + 1.

$$MPI_j(t) = \left[\frac{M_j^{(t)s*}(t+1) M_j^{(t+1)s*}(t+1)}{M_j^{(t)s*}(t) M_j^{(t+1)s*}(t)}\right]^{\frac{1}{2}}, \quad \text{for all j.} \tag{19}$$

In *Eq. (19)*, the whole system is considered as a DMU, and it explains that productivity decreases if the value of the index is lower than one, stays unchanged if it equals one, and amends if it is larger than one. Based on the value of MPI, the trend of productivity is as follows:

I. $MPI_j(t) > 1$ shows an increase or progress in the productivity of the DMU_0.

II. $MPI_j(t) = 1$ reflects no change in productivity during the two periods of the DMU_0.

III. $MPI_j(t) < 1$ reveals a regress in the productivity of DMU_0.

The MPI can be divided to scale the change in efficiency and the shift of the frontier between both periods as follows:

$$MPI_j(t) = \frac{M_j^{(t+1)s*}(t+1)}{M_j^{(t)s*}(t)} \times \left[\frac{M_j^{(t)s*}(t+1) M_j^{(t)s*}(t)}{M_j^{(t+1)s*}(t+1) M_j^{(t+1)s*}(t)}\right]^{\frac{1}{2}}, \quad \text{for all j.} \tag{20}$$

The first right-hand side term computes the technical efficiency variation taking place between both periods, while the second term scales the corresponding shift in the technology frontier and is generally mentioned as a technical variation. Given the efficiency scores gained from *Models (7)-(10)*, the MPI for the whole process of the j^{th}DMU can be computed utilizing *Eq. (20)*. Similarly, the efficiency scores deduced from *Models (11)-(14)* can be applied to compute the MPI for the first stage of the j^{th} DMU utilizing *Eq. (21)*:

$$MPI_{1j}(t) = \frac{M_{1j}^{(t+1)s*}(t+1)}{M_{1j}^{(t)s*}(t)} \times \left[\frac{M_{1j}^{(t)s*}(t+1) M_{1j}^{(t)s*}(t)}{M_{1j}^{(t+1)s*}(t+1) M_{1j}^{(t+1)s*}(t)}\right]^{\frac{1}{2}}, \quad \text{for all j.} \tag{21}$$

Ultimately, the efficiency scores resumed from *Models (15)-(17)* can be applied to compute the MPI for the second stage of the j^{th} DMU utilizing *Eq. (22)*:

$$MPI_{2j}(t) = \frac{M_{2j}^{(t+1)s*}(t+1)}{M_{2j}^{(t)s*}(t)} \times \left[\frac{M_{2j}^{(t)s*}(t+1) M_{2j}^{(t)s*}(t)}{M_{2j}^{(t+1)s*}(t+1) M_{2j}^{(t+1)s*}(t)}\right]^{\frac{1}{2}}, \quad \text{for all j.} \tag{22}$$

5 | Case Study

More than a century ago, the growth of the Iranian oilfield industry began. The intricate processes and structures of manufacturing refineries, which include filtration units, catalytic conversion, and the refinement of liquid gas and oil, aim to decrease the energy expended. Also, the intricacy index of each Iranian oilfield is different; the operating units dealing with the corresponding refining and filtration activities face different necessities than other units in other oilfields. On the other hand, in the operation of real oil generation, freshwater is required, so a large amount of wastewater as undesirable output is produced. Thus, in oilfields, any decrease in water consumption means a reduction in oil generation. In the situation of restricted resources, it is essential to improve the efficiency of oil generation and wastewater treatment. Moreover, in accordance with the supportable expansion program, it is required to reuse wastewater to warrant the water reserve in the water-deficient regions [40]. Thus, the wastewater here behaves as a feedback variable, which improves the efficiency of the total oilfield system. Further, uncertainty in the number of wells is an essential issue in the development of oilfields. Therefore, every decision needs to take into account all the uncertainties in all stages of field development. Here, we evaluate the offered models under discrete scenarios provided by the oilfield system analyzers (i.e., s_1 =Pessimistic, s_2 =Optimistic). According to Snyder and Daskin [41], we assume that all scenarios have equiprobable that is q^s =0.5. Further, we consider five inputs in stage 1: the number of generating wells, cost of oil, cost of water, water increasing rate, and reusable water; desirable outputs and undesirable outputs are actual oil generation and incremental oil generation, respectively. In stage 2, undesirable wastewater is refined by applying inputs such as operating costs, consumption costs, construction expenditure, and hydrocarbons; therefore, hydrocarbon removal rates and the quantity of reusable refined wastewater are desirable outputs, and unrefined wastewater is undesirable output. All data have been gathered over a period of two years (2020-2021) through consultations with experts. The inputs and outputs data for the years 2020 and 2021 are listed in *Tables 4* and *5*, respectively. The results gained are displayed in two main sections. In the first section, the whole efficiency, including those of the first and second stages, is prepared. In the second section, the MPI is computed both for the entire process and each one of its stages.

Table 4. The data set of three scenarios for 11 oilfields (2020).

DMUs	x_1^1		x_2^1		x_3^1		x_4^1		y_1^1		y_2^1		z_1
	s_1	s_2	s_1	s_2	s_1	s_2	s_1	s_2	s_1	s_2	s_1	s_2	s_1
1	75000	90000	0.498	0.510	92.22	121.57	67.45	60.8	510.1	638.1	102.0	68.0	433.0
2	48000	84000	0.442	0.453	73.36	132.05	85.5	95	797.1	997.1	72.0	96.0	344.4
3	52000	87000	0.434	0.462	113.18	117.38	55.1	64.6	542.0	678.0	87.0	116.0	531.3
4	4000	74000	0.533	0.546	209.60	138.34	30.4	27.55	231.2	289.2	150.0	200.0	984.0
5	35000	63000	0.410	0.420	79.65	125.76	78.85	82.65	693.5	867.5	85.5	114.0	373.9
6	30000	90000	0.342	0.351	127.86	117.38	48.45	40.85	342.8	428.8	87.0	116.0	600.2
7	18000	90000	0.513	0.525	138.34	119.47	45.6	40.85	342.8	428.8	96.0	128.0	649.4
8	6000	95000	0.368	0.377	119.47	117.38	52.25	52.25	438.4	548.4	90.0	120.0	560.9
9	46000	95000	0.523	0.536	85.94	123.66	72.2	64.6	542.0	678.0	52.5	70.0	403.4
10	23700	90000	0.564	0.577	113.18	117.38	56.05	46.55	390.6	488.6	82.5	110.0	531.3
11	12000	84000	0.318	0.326	88.03	121.57	71.25	58.9	494.2	618.2	61.5	82.0	413.3

Table 5. The data set of three scenarios for 11 oilfields (2021).

DMUs	x_1^1		x_2^1		x_3^1		x_4^1		y_1^1		y_2^1		z_1
	s_1	s_2	s_1	s_2	s_1	s_2	s_1	s_2	s_1	s_2	s_1	s_2	s_1
1	75750	90900	0.503	0.515	93.1	122.8	68.1	61.4	515.2	644.5	103.0	68.7	437.3
2	48480	84840	0.446	0.458	74.1	133.4	86.4	96.0	805.1	1007.1	72.7	97.0	347.8
3	52520	87870	0.438	0.467	114.3	118.6	55.7	65.2	547.4	684.8	87.9	117.2	536.6
4	4040	74740	0.538	0.551	211.7	139.7	30.7	27.8	233.5	292.1	151.5	202.0	993.8
5	35350	63630	0.414	0.424	80.4	127.0	79.6	83.5	700.4	876.2	86.4	115.1	377.6
6	30300	90900	0.345	0.355	129.1	118.6	48.9	41.3	346.2	433.1	87.9	117.2	606.2
7	18180	90900	0.518	0.530	139.7	120.7	46.1	41.3	346.2	433.1	97.0	129.3	655.9
8	6060	95950	0.372	0.381	120.7	118.6	52.8	52.8	442.8	553.9	90.9	121.2	566.5
9	46460	95950	0.528	0.541	86.8	124.9	72.9	65.2	547.4	684.8	53.0	70.7	407.4
10	23937	90900	0.570	0.583	114.3	118.6	56.6	47.0	394.5	493.5	83.3	111.1	536.6
11	12120	84840	0.321	0.329	88.9	122.8	72.0	59.5	499.1	624.4	62.1	82.8	417.4

First, using *Model (7)*, we get the ideal efficiency score of each DMU based on each scenario in both stages and overall. The related results of the ideal efficiency score matching the amounts specified in each scenario in the 2020 and 2021 years are reported in the columns of *Table 6*.

Table 6. Ideal efficiency scores in two scenarios of Model (7).

DMUs	s_1 (2020)	s_2 (2020)	s_1 (2021)	s_2 (2021)	s_1 (2020)	s_2 (2020)	s_1 (2021)	s_2 (2021)	s_1 (2020)	s_2 (2020)	s_1 (2021)	s_2 (2021)
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	0.770	1.000	1.000	1.000	0.257	1.000	1.000	1.000	0.128	1.000	1.000	1.000
3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
5	0.728	0.945	1.000	1.000	0.243	0.315	1.000	1.000	0.121	0.158	1.000	1.000
6	1.000	1.000	0.977	0.893	1.000	1.000	0.326	0.298	1.000	1.000	0.163	0.149
7	1.000	1.000	0.832	1.000	1.000	1.000	0.277	1.000	1.000	1.000	0.139	1.000
8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
9	0.955	1.000	1.000	1.000	0.318	1.000	1.000	1.000	0.159	1.000	1.000	1.000
10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
11	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

With respect to *Table 6*, the efficiency scores in *Model (7)* for most DMUs are equal to one that is 72.7% and 81.8% of the total oilfields in the first scenario from 2020 and 2021 years from right to left, respectively. As well, most DMUs gained an efficiency score of one that is 90.9% in the second scenario of total oilfields in both periods. Subsequently, we solved *Models (7)* and *(10)* to gain the overall efficiency scores for different p-values in each scenario in the 2020 and 2021 years. Then, we solved *Models (13)-(15),* and *(18)* to get the efficiency scores of the first and second stages, respectively, which are presented in *Tables 7* and *8*. Also, the efficiency scores of the overall process with its corresponding first and second stages in *Fig. 3* are illustrated. As can be seen, the abovementioned models give infeasible results for some DMUs when $p \leq 0.49$, and we do not report those here.

Table 7. The results of the overall and the stages efficiency scores in 2020.

Efficiency	**Overall**				**First Stage**				**Second Stage**			
P-value DMUs	0.49	0.50	0.51	0.52	0.49	0.50	0.51	0.52	0.49	0.50	0.51	0.52
1	0.718	0.887	0.888	0.889	0.976	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	0.854	1.000	1.000	1.000	0.999	1.000	1.000	1.000	0.855	1.000	1.000	1.000
3	0.832	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
4	0.649	0.671	0.671	0.672	0.722	0.735	0.734	0.735	0.707	0.660	0.657	0.659
5	INF	0.832	0.832	0.835	INF	0.967	0.967	0.969	0.838	0.748	0.707	0.707
6	0.604	0.681	0.681	0.681	0.714	0.716	0.717	0.719	0.763	0.739	0.763	0.764
7	0.639	0.821	0.821	0.822	0.693	0.727	0.727	0.729	0.958	0.830	0.794	0.794
8	0.743	0.757	0.757	0.758	0.693	0.733	0.733	0.733	0.599	0.559	0.545	0.547
9	INF	0.852	0.852	0.853	0.653	0.897	0.898	0.898	0.789	0.735	0.719	0.719
10	0.745	0.999	0.999	0.999	0.749	1.000	1.000	1.000	INF	0.674	0.709	0.711
11	0.887	0.997	0.997	0.997	INF	0.831	0.831	0.832	1.000	1.000	1.000	1.000

Table 8. The results of the overall and the stages efficiency scores in 2021.

	Overall Efficiency				**First Stage**				**Second Stage**			
P-Value DMUs	0.49	0.50	0.51	0.52	0.49	0.50	0.51	0.52	0.49	0.50	0.51	0.52
1	0.594	0.619	0.619	0.619	0.732	0.724	1.000	1.000	0.238	0.238	0.238	0.239
2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3	0.748	0.858	0.858	0.859	0.515	0.594	0.831	0.831	INF	0.845	0.846	0.845
4	0.468	0.518	0.519	0.519	0.587	0.662	0.663	0.664	0.300	0.301	0.302	0.302
5	INF	0.617	0.617	0.617	INF	0.774	0.721	0.722	0.448	0.466	0.466	0.466
6	0.758	0.857	0.859	0.859	0.577	0.627	0.531	0.532	1.000	1.000	1.000	1.000
7	0.600	0.789	0.790	0.791	0.577	0.588	0.793	0.793	0.830	0.851	0.851	0.851
8	0.736	0.769	0.769	0.769	0.532	0.512	0.535	0.536	0.746	0.804	0.805	0.805

Table 8. Continued.

P-Value DMUs	Overall Efficiency 0.49	0.50	0.51	0.52	First Stage 0.49	0.50	0.51	0.52	Second Stage 0.49	0.50	0.51	0.52
9	INF	0.852	0.853	0.854	0.942	0.929	0.723	0.723	0.733	0.896	0.897	0.897
10	0.687	0.714	0.716	0.716	0.725	0.738	0.713	0.717	0.670	0.528	0.529	0.529
11	0.604	0.851	0.853	0.853	0.535	0.529	0.682	0.683	0.763	0.871	0.872	0.871

As mentioned before, for small values of p, the proposed models give infeasible results in some scenarios. In this study, on the one hand, when p ≤ 0.49 with respect to the results, our models give infeasible results for some DMUs. As the p-value enhances, the efficiency scores are set better, and the number of infeasible DMUs gradually reduces, and we see feasible results. On the other hand, for p≥ 0.50, the efficiency scores remain constant. So, we do not carry on and stop it for the other p-values. Thus, here, we only consider p≥ 0.50 and do not report the results of p< 0.49. For example, in *Table 7*, by increasing the p-value from 0.49 to 0.50, the efficiency score of DMU #5 and DMU #9 shifts. This shift also can be seen in some DMUs of the first and the second stages. It is noted that *Models (7)-(19)* maximize the expected efficiency scores of DMUs in each scenario, while p-robust constraints control the respective variation between their efficiency scores produced by the model and ideal efficiency under each scenario. Further, *Tables 9-11* show the results of the efficiency scores of DMUs in each scenario in the years 2020 and 2021 with *Models (8)*, *(9)*, *(12)*, *(13)*, *(16)*, and *(17)* for p= 0.50.

Table 9. Overall efficiency scores in both periods for $p = 0.50$.

DMUs	1	2	3	4	5	6	7	8	9	10	11
$M_0^{(t)s*}(t)$	0.887	1.000	1.000	0.671	0.832	0.681	0.821	0.757	0.852	0.999	0.997
$M_0^{(t+1)s*}(t+1)$	0.619	1.000	0.858	0.518	0.617	0.857	0.789	0.769	0.852	0.714	0.851
$M_0^{(t)s*}(t+1)$	0.351	0.249	0.489	0.490	0.322	0.491	0.179	0.395	0.467	0.500	0.242
$M_0^{(t+1)s*}(t)$	0.463	0.182	0.306	0.143	0.219	0.084	0.169	0.225	0.233	0.047	0.231

As can be seen in *Table 9*, DMU2 is efficient in 2020 and 2021, and the efficiency ratio is constant over the two consecutive periods; however, its performance is not efficient in 2020 in comparison to 2021. Also, DMU9 is not efficient in 2020 and 2021, but this ratio is constant over the two consecutive periods; however, its performance is not efficient in 2020 in comparison to 2021. Unlike DMU6 and DMU8, the efficiency values of DMU1, DMU3, DMU4, DMU5, DMU7, DMU10, and DMU11 are greater in 2020 proportionate to 2021, compared to 2021 proportionate to 2020. As DMU3 is efficient in 2020 and inefficient in 2021, the ratio of its efficiency values is less than 1 in the two sequential periods (2020 and 2021), indicating that their annual efficiency is smaller in 2021 proportionate to 2020 than vice versa.

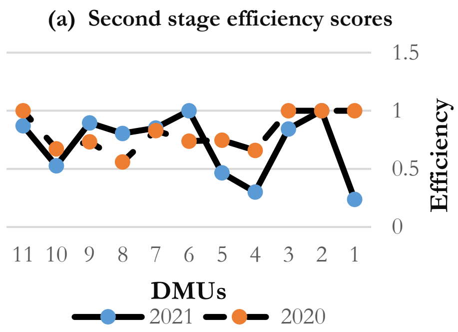
Table 10. The efficiency scores of the first stage in both periods for $p = 0.50$.

DMUs	1	2	3	4	5	6	7	8	9	10	11
$M_{10}^{(t)s*}(t)$	1.000	1.000	1.000	0.735	0.967	0.716	0.727	0.733	0.897	1.000	0.831
$M_{10}^{(t+1)s*}(t+1)$	0.724	1.000	0.594	0.662	0.617	0.627	0.588	0.512	0.929	0.738	0.529
$M_{10}^{(t)s*}(t+1)$	1.000	1.000	0.426	1.000	1.000	0.089	0.233	1.000	1.000	0.276	1.000
$M_{10}^{(t+1)s*}(t)$	0.080	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.323	1.000	0.182

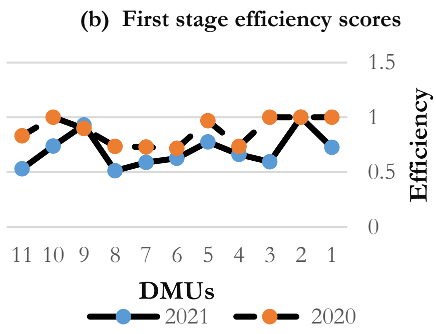
Table 11. The efficiency scores of the second stage in both periods for $p = 0.50$.

DMUs	1	2	3	4	5	6	7	8	9	10	11
$M_{20}^{(t)s*}(t)$	1.000	1.000	1.000	0.660	0.748	0.739	0.830	0.559	0.735	0.674	1.000
$M_{20}^{(t+1)s*}(t+1)$	0.238	1.000	0.845	0.301	0.466	1.000	0.851	0.804	0.896	0.528	0.871
$M_{20}^{(t)s*}(t+1)$	1.489	0.536	0.064	0.503	0.495	1.000	1.000	0.038	1.000	1.000	0.244
$M_{20}^{(t+1)s*}(t)$	0.003	0.135	0.061	0.012	0.367	1.000	1.000	1.000	0.048	0.167	1.000

Moreover, *Fig. 3* displays that the overall efficiency score of DMU2 is the highest one among all DMUs in both years (i.e., 2020 and 2021), while DMU 4 displays the lowest overall efficiency score in both years. The results vary when considering the efficiency scores, which are proposed in the first stage of *Table 10* and *Fig. 3*. In this case, the DMU2 shows the highest efficiency score when considering both years, while the DMU6 in the first stage and DMU8 in the second stage have the lowest efficiency in periods. Finally, DMU2 depicts the highest efficiency score through the second stage in both years, with DMU1 and DMU8 displaying the lowest efficiency score in the first and the second stage periods, respectively.



a.



b.

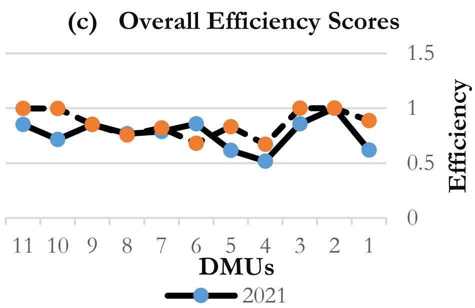


Fig. 3. The efficiency of the overall and the stages throughout 2020 to 2021; a. second stage efficieny scores, first stage efficieny scores, overal efficieny scores.

In the sequel, MPIs are computed according to *Eqs. (21)-(23)* all, together with the relative models accounting for the efficiency of the overall process, the first and the second stages. The MPI includes two primary elements, that is, the change in technology change and technical efficiency taking place during the two periods of 2020 and 2021. *Table 12* shows the MPI together with the efficiency and technology changes for the overall procedure, moreover, its first and second stages. Also, the efficiency values of the first set of columns display that DMU2, DMU3, DMU4, DMU6, DMU8, DMU9, and DMU10 have improved the MPI of their total processes from 2020 to 2021, while all the other oilfields have stood a worsening. It should be noted that the mean MPI for the total process presents progress from 2020 to 2021. It is noted that DMUs differ in their respective efficiency and technology changes, with DMU10 displaying a nearly inverted behavior in both efficiencies.

Table 12. MPI for the overall process, the first and the second stages.

	Overall Process 2020-2021			First Stage 2020-2021			Second Stage 2020-2021		
DMUs	EC	TC	MPI	EC	TC	MPI	EC	TC	MPI
1	0.619	1.042	0.645	0.724	4.155	3.008	0.238	45.667	10.869
2	1.000	1.170	1.170	1.000	1.000	1.000	1.000	1.993	1.993
3	0.858	1.365	1.171	0.594	0.847	0.503	0.845	1.114	0.941
4	0.518	2.107	1.091	0.901	1.054	0.950	0.456	9.587	4.372
5	0.617	1.408	0.869	0.638	1.252	0.799	0.623	1.471	0.916
6	0.857	2.155	1.847	0.876	0.319	0.279	1.353	0.860	1.164
7	0.789	0.492	0.388	0.809	0.537	0.434	1.025	0.988	1.013
8	0.769	1.315	1.011	0.698	1.197	0.836	1.438	0.163	0.234
9	0.852	1.416	1.206	1.036	1.729	1.791	1.219	4.134	5.039
10	0.714	3.858	2.755	0.738	0.612	0.452	0.783	2.765	2.165
11	0.851	1.108	0.943	0.637	2.938	1.872	0.871	0.529	0.461
Average	0.768	1.585	1.191	0.786	1.422	1.084	0.896	6.297	2.652

Table 12 displays the average growth rate of the total productivity of DMUs for the overall process; the first and the second stages are, respectively, 1.191, 1.084, and 2.652, representing an average increase in total productivity over the two periods of 2020 and 2021. Such an increase rises from the increased mean of TC. In *Table 12*, four oilfields (DMU1, DMU5, DMU7, and DMU11), out of 11 oilfields, sustained a regress in total productivity index between 2020 and 2021, and the seven other oilfields had improved productivity. It is noted that the TC values less than 1 indicate the regress of the technology, and the values bigger than 1 indicate the progress of the technology. As well, EC values bigger than 1 indicate an increase in performance, and EC values less than 1 show a decrease in performance. Now, we consider the efficiency and technology values gained for the first and second stages in the last two sets of columns. The values gained for the first stage show that DMU1, DMU2, DMU9, and DMU11 have improved their MPI over the 2020 and 2021 periods, while the other DMUs have drained a worsening.

Similarly to the total efficiency, DMUs differ in their respective efficiency and technology changes through the first stage over the period assumed, with DMU6 experiencing the worst efficiency change. The last set of columns represents the fact that the MPI of the second stage has improved over the 2020 and 2021 periods in all DMUs except for DMU3, DMU5, DMU8, and DMU11. Also, DMUs differ in their respective technology and efficiency changes, with DMU1 and DMU4 performing an entirely opposite behavior in both efficiencies and DMU8 exhibiting the worst technology change.

6 | Conclusion

We designed a model using DEA and MPI to evaluate the efficiency of a two-stage network. The presented models have been applied to get the efficiency and productivity of the whole process as the first and second stages of the network system. The presented approach not only specifies patterns of productivity change and gives a new interpretation along with the managerial implication of each Malmquist ingredient but also identifies strategic orientations of DMUs in past periods for suitable choices in future periods. This approach has been handled to evaluate the proficiency of 11 oilfields in the Persian Gulf over the 2020 and 2021 periods. The results gained are helpful to better the perception of Iranian oilfields and their internal structures. These results show the variations in efficiency and productivity of the factors of production, which, simultaneously, has permitted us to display that the investments are inadequate to raise the growth of the relative technology levels. It noted that the rate of degradation of capital types of equipment in the oil region is exceptionally high, confirming the necessity to increase investment to substitute the underestimated assets.

Conflict of Interest Disclosure

The authors declare they have no competing interests as defined by the journal, or other interests that might be perceived to influence the results reported in this paper.

Data Access

Anonymized data can be requested from the corresponding author following journal data sharing policies.

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