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A Modified Model to Find the Most Efficient Decision-Making Unit

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Abstract

The use of Data-Envelopment Analysis (DEA) to determine the most efficient Decision Making Unit (DMU) has drawn attention in the literature. For some applications of DEA, decision-makers may only want to identify the most efficient DMU rather than determine the efficiencies of all possible DMUs. In this study, we present a modified model based on the model in Özsoy et al. [1], A simplistic approach without epsilon to choose the most efficient unit in DEA. Expert systems with applications, 2021. 168: p. 114472] to identify the most efficient DMU. The proposed model, with low complexity with respect to the model in [1], finds only one DMU with an efficiency score greater than one, while all others receive scores strictly less than one. This structure enhances the model's ability to fully rank all units. To demonstrate its effectiveness and compare it with two well-known models, the proposed model is applied to two real-world examples from the literature. The results show the appropriate performance of the proposed model in identifying the most efficient units and full ranking of the units.

Keywords: Data envelopment analysis, Most efficient decision making unit, Mixed integer linear programming, Ranking.

1 | Introduction

Data Envelopment Analysis (DEA) is a mathematical approach introduced by Charnes et al. [2] to assess the relative efficiency of a homogeneous group of Decision-Making Units (DMUs). DEA successfully divides DMUs into two categories: efficient DMUs and inefficient DMUs. In some cases, the decision-maker must select only one DMU among efficient DMUs, which is called the most efficient DMU. Therefore, several studies have been done to find the most efficient unit in DEA. To evaluate the most efficient DMU in Advanced Manufacturing Technology (AMT), Karsak and Ahiska [3] proposed an integrated Multi-Criteria Decision-Making (MCDM) DEA model. To overcome the convergence of the proposed model in [3], Amin et al. [4] modified and improved it. Amin and Toloo [5] proposed a new Mixed Integer Linear Programming (MILP) model based on CSW to find the most efficient unit. For selecting the

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most BCC-efficient DMU, Toloo and Nalchigar [6] extended this model into a Variable Returns to Scale (VRS) situation. Amin et al. [4], Amin [7] introduced a new Mixed Integer Non-Linear Programming (MINLP) model for overcoming some drawbacks of previous MILP models. Although their models can determine the most efficient unit, they are non-linear and therefore difficult to solve.

Toloo et al. [8] revealed that the problem of finding the most significant association rule by considering multiple criteria in data mining is an important task and designed an algorithm for prioritizing association rules. This algorithm has some drawbacks that are mentioned and improved by Toloo and Nalchigar [9]. By maximizing the minimum possible distance between a selected unit and the next-ranked unit, Foroughi [10] proposed a new MILP model to find the most efficient unit. This approach can also be extended to rank all extreme efficient DMUs. By removing additional constraints in Foroughi's model, Wang and Jiang [11] proposed a new model to identify the most efficient DMU, which is less complex than Foroughi's model. Toloo [12] proposed a new MILP model for selecting the most efficient DMU without explicit input and utilized this model to determine the best efficient professional tennis player. Toloo [13] excluded the non-Archimedean epsilon and proposed a new model with fewer computations to find the most efficient DMU. Toloo [14] showed that in the supply chain, the selection and full-ranking of suppliers with imprecise data is a very important issue. Using the CSW method, Toloo [15] introduced a new minimax MILP model for selecting the most efficient DMU. Lam [16] introduced a new MILP model similar to that of the super-efficiency model for directly discovering the most efficient DMU.

Salahi and Toloo [17] illustrated that Lam's model may be infeasible, and they proposed a modified model to cope with this issue. Toloo [18] proposed a method for finding the most cost-efficient DMU by utilizing the proposed approach in [19] when the prices are fixed and known. Toloo and Salahi [20] developed a new two-step MINP model involving the epsilon, which identifies a single efficient DMU whose efficiency score is strictly greater than one. Both non-linear models can be turned into linear models. Based on the proposed model in [20], Özsoy et al. [1] proposed a mixed integer programming model without epsilon with one step for selecting the most efficient unit. This model, with fewer constraints than the model in [20], determines exactly one DMU as the most efficient, with an efficiency score greater than one, while the other DMUs have efficiency scores strictly below one.

Ebrahimi et al. [21] analyzed the two-step method proposed by [16], [17] to identify the most efficient units. They mathematically proved that the first-step model in [17] is sufficient to determine the best DMU, rendering the second step redundant. They improved the first-step model by proposing a modified version, demonstrating that it could identify the best DMU with considerably lower computational effort. Noori et al. [22] explored the link between the most efficient and extremely efficient units. Their findings showed that an extremely efficient unit can also be considered the most efficient, and the reverse holds true as well. This implies that the defining properties of extremely efficient units are essentially the same as those of the most efficient units. The contribution of this study is to develop a new discriminative MILP model that identifies a single efficient DMU whose efficiency score is strictly greater than one. This formulation enhances the model's discriminatory power, simplifies its structure, and reduces the number of constraints—leading to better computational efficiency. A comparative analysis with two famous models, using benchmark DEA case studies, highlights the superior performance of the proposed method. The rest of the paper is structured as follows. Section 2 gives a brief overview of two well-known models for identifying the most efficient DMU. In Section 3, we introduce our proposed MILP model and explain how it works. Section 4 presents two numerical examples to illustrate how the model can be applied in practice and to highlight its effectiveness. Finally, Section 5 wraps up the paper with concluding remarks and suggestions for future research.

2 | The Two Well-Known Models to Find the Most Efficient Unit

The two well-known models related to finding the most efficient DMU were reviewed in this section.

Suppose there are n DMUs to be evaluated, $DMU_j (j = 1,2,...,n)$ each using m inputs to produce s outputs. Let $x_{ij} (i = 1,2,...,m)$ and $y_{rj} (r = 1,2,...,s)$ represent the input and output values of DMU_j, respectively.

2.1 | The Wang and Jiang's Model

Wang and Jiang [11] proposed the following MILP model for finding the most CCR-efficient DMU under CRS.

$$
\begin{aligned}
&\text{Min} \sum_{i=1}^{m} v_i \left(\sum_{j=}^{n} x_{ij} \right) - \sum_{i=1}^{m} u_r \left(\sum_{j=}^{n} y_{ij} \right) \\
&\text{s.t.} \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le I_j, \; j = 1,2,...,n, \\
&\qquad \sum_{j=1}^{n} I_j = 1, \\
&\qquad u_r \ge l_r^u, \; r = 1,2,...,s, \\
&\qquad v_i \ge l_i^v, \; i = 1,2,...,m, \\
&\qquad I_j \in \{0,1\}, \quad j = 1,2,....,n,
\end{aligned} \tag{1}
$$

where $l_r^u = ((m+s)\max_j\{y_{rj}\})^{-1}$ and $l_i^v = ((m+s)\max_j\{x_{ij}\})^{-1}$ lower bounds borrowed from [23]. *Model (1)* is feasible, and its objective is to maximize the overall efficiency of all of the DMUs. In this model, if $I_p^* = 1$ then $\sum_{r=1}^{s} u_r y_{rp} - \sum_{i=1}^{m} v_i x_{ip} \le 1$, hence, *Model (1)* allows efficiency value of DMU_p to be larger than one, and, on the other hand, for $I_j^* = 0 (j \ne p)$, the efficiency value of DMU_j is less than or equal to one due to the constraint $\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0$. So, in *Model (1),* DMU_p is determined as the most efficient DMU if and only if $I_p^* = 1$.

2.2 | The Özsoy, Örkcü's Model

Inspired by the Toloo and Salahi [20] model, Özsoy et al. [1] presented a new single-stage MINLP model to find the most efficient DMU as follows:

$$
\begin{aligned}
&h^* = \text{Max} \quad h \\
&\text{s.t.} \\
&\qquad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le MI_j - h(1 - I_j), \quad j = 1,2,...,n, \\
&\qquad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \ge hI_j - M(1 - I_j), \quad j = 1,2,...,n, \\
&\qquad \sum_{i=1}^{n} I_j = 1, \\
&\qquad I_j \in \{0,1\}, \; j = 1,2,...,n, \\
&\qquad u_r \ge ((m+s)\max_j\{y_{rj}\})^{-1}, \quad r = 1,2,...,s, \\
&\qquad v_i \ge ((m+s)\max_j\{x_{ij}\})^{-1}, \quad i = 1,2,...,m,
\end{aligned} \tag{2}
$$

where M is a large positive number. The minimum possible interval between the first two top-ranking DMUs is [-h*, h*], where h* is strictly positive. *Model (2)* identifies exactly one DMU ($DMU_p, I_p^* = 1$) as the most efficient, with an efficiency score greater than one, while all other DMUs $(DMU_j, j \ne p)$ have efficiency scores strictly less than one. *Model (2),* by using the continuous variable $z_j = hI_j$ and adding the following constraints, is transformed into an MILP model [1], [20].

$$
\begin{aligned}
&0 \le z_j \le MI_j \\
&z_j \le h \le z_j + M(1 - I_j).
\end{aligned}
$$

3 | The Proposed Model

Models (1) and *(2)* have inspired us to propose the following model for finding the most efficient DMU:

$$\begin{aligned}
&\text{Max} \quad \sum_{r=1}^{s} u_r \sum_{j=1}^{n} y_{rj} - \sum_{i=1}^{m} v_i \sum_{j=1}^{n} x_{ij} \\
&\text{s.t.} \\
&\quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le MI_j - (1 - I_j), \quad j = 1,2,\dots,n, \\
&\quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \ge I_j - M(1 - I_j), \quad j = 1,2,\dots,n, \\
&\quad \sum_{i=1}^{n} I_j = 1, \\
&\quad I_j \in \{0,1\}, \; j = 1,2,\dots,n, \\
&\quad u_r \ge ((m+s)\max_j\{y_{rj}\})^{-1}, \quad r = 1,2,\dots,s, \\
&\quad v_i \ge ((m+s)\max_j\{x_{ij}\})^{-1}, \quad i = 1,2,\dots,m.
\end{aligned} \tag{3}$$

Theorem 1. The *Model (3)* is feasible. Let $u_r^*(r = 1,2,\dots,s), v_i^*(i = 1,2,\dots,m), h^*, I_j^*(j = 1,2,\dots n)$ be the optimal solution in *Model (2)*. So we have

$$h^* I_j^* - M(1 - I_j^*) \le \sum_{r=1}^{s} u_r^* y_{rj} - \sum_{i=1}^{m} v_i^* x_{ij} \le M I_j^* - h^*(1 - I_j^*), \quad j = 1,2,\dots,n.$$

As mentioned in the previous section, $h^* > 0$, thus

$$I_j^* - \frac{M}{h^*}(1 - I_j^*) \le \sum_{r=1}^{s} \frac{u_r^*}{h^*} y_{rj} - \sum_{i=1}^{m} \frac{v_i^*}{h^*} x_{ij} \le \frac{M}{h^*} I_j^* - (1 - I_j^*), \quad j = 1,2,\dots,n.$$

$\bar{u}_r = \frac{u_r^*}{h^*}(r = 1,2,\dots,s), \bar{v}_i = \frac{v_i^*}{h^*}(i = 1,2,\dots,m), I_j^*(j = 1,2,\dots n)$ is a feasible solution for *Eq. (3)*. Let $\left(u_r^*(r = 1,2,\dots,s), v_i^*(i = 1,2,\dots,m), I_j^*(j = 1,2,\dots,n)\right)$ be an optimal solution of *Model (3)*. If $I_p^* = 1$, then $s_p^* = 0$, so $1 \le \sum_{r=1}^{s} u_r y_{rp} - \sum_{i=1}^{m} v_i x_{ip} \le M$. This allows the efficiency DMU_p to be greater than one. On the other hand, if $I_j^* = 0 (j = 1,2,\dots,n; j \ne p)$, then $-M \le \sum_{r=1}^{s} u_r y_{rp} - \sum_{i=1}^{m} v_i x_{ip} \le -1$. This guarantees that the efficiencies of the other DMUs are less than one.

4 | Numerical Example

In this example, we use real data from nineteen Facility Layout Designs (FLDs) studied by Ertay and Ruan [24]. Each FLD consumes two inputs, cost (x_1) and adjacency score (x_2), to produce shape ratio (y_1), flexibility (y_2), quality (y_3), and hand-carry utility (y_4) as four outputs. The data appear in columns two through seven of *Table 1*. In this example, we use $M = 100$. *Table 1* presents the outcomes of *Models (1)-(3)*. *Models (1)-(3)* consistently identify FLD10 as the most efficient design. However, *Model (1)* cannot fully distinguish among all DMUs; for instance, FLD3 and FLD12 receive the same rank. In contrast, *Models (2)* and *(3)* are capable of ranking all DMUs effectively. It is also worth noting that FLD13 is identified as the least efficient DMU across all models. As shown in *Table 2*, the correlation coefficient between the proposed model and the model by Özsoy et al. [1] is 0.78245614 (7.51884977459034e-05). This result indicates that the two models are statistically concordant at the significance level ($\alpha = 0.05$). Moreover, the proposed model, with fewer constraints, successfully ranks all FLDs in a single step. *Fig. 1* shows the efficiency score of *Models (1)-(3)*.

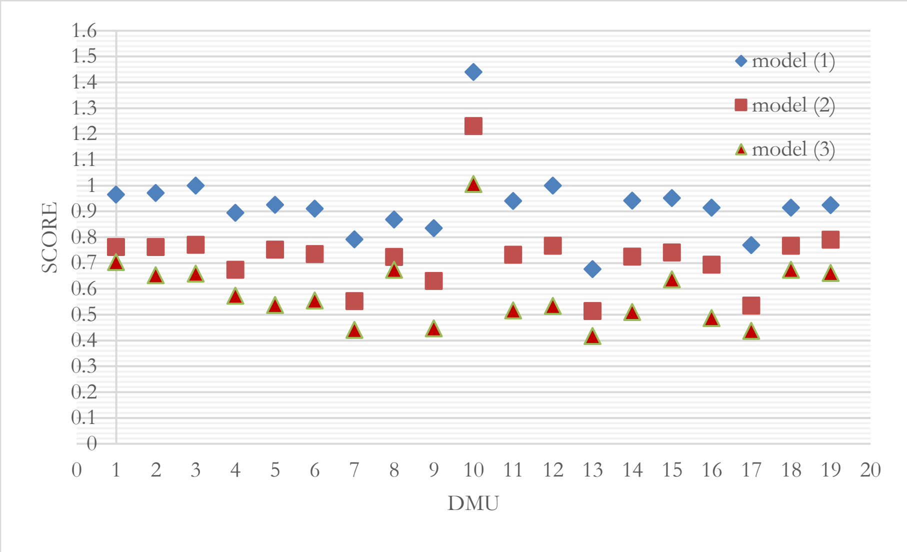

Fig. 1. Illustrative comparison between the efficiency scores of Models (1)-(3).

Table 1. Data set for 19 FLDs and efficiency of FLDs by different models.

DMUs	Inputs		Outputs				Wang and Jiang [11] Model (1)	Özsoy et al. [1] Model (2)	Proposed Model Model (3)
	x_1	x_2	y_1	y_2	y_3	y_4			
FLD1	20,309.56	6,405	0.4697	0.0113	0.041	30.89	0.964891 (5)	0.761219 (7)	0.703336(2)
FLD2	20,411.22	5,393	0.438	0.0337	0.0484	31.34	0.971531 (4)	0.761527 (6)	0.653074(7)
FLD3	20,280.28	5,294	0.4392	0.0308	0.0653	30.26	1 (2)	0.770702 (3)	0.659215(6)
FLD4	20,053.20	4,450	0.3776	0.0245	0.0638	28.03	0.894522 (14)	0.673692 (15)	0.57332(9)
FLD5	19,998.75	4,370	0.3526	0.0856	0.0484	25.43	0.925330 (9)	0.751551 (8)	0.537374(11)
FLD6	20,193.68	4,393	0.3674	0.0717	0.0361	29.11	0.910794 (13)	0.734339 (10)	0.554245(10)
FLD7	19,779.73	2,862	0.2854	0.0245	0.0846	25.29	0.790849 (17)	0.552031 (17)	0.439958(17)
FLD8	19,831	5,473	0.4398	0.0113	0.0125	24.8	0.868210 (15)	0.723427 (13)	0.674081(3)
FLD9	19,608.43	5,161	0.2868	0.0674	0.0724	24.45	0.834482 (16)	0.630595 (16)	0.446235(16)
FLD10	20,038.10	6,078	0.6624	0.0856	0.0653	26.45	1.440321 (1)	1.230623 (1)	1.005713(1)
FLD11	20,330.68	4,516	0.3437	0.0856	0.0638	29.46	0.940190 (8)	0.732256 (11)	0.515594(13)
FLD12	20,155.09	3,702	0.3526	0.0856	0.0846	28.07	1 (2)	0.766601 (5)	0.533705(12)
FLD13	19,641.86	5,726	0.269	0.0337	0.0361	24.58	0.675683 (19)	0.513299 (19)	0.417161(19)
FLD14	20,575.67	4,639	0.3441	0.0856	0.0638	32.2	0.941034 (7)	0.723855 (12)	0.510119(14)
FLD15	20,687.50	5,646	0.4326	0.0337	0.0452	33.21	0.951281 (6)	0.740819 (9)	0.63644(8)
FLD16	20,779.75	5,507	0.3312	0.0856	0.0653	33.6	0.913958 (11)	0.693781 (14)	0.4863(15)
FLD17	19,853.38	3,912	0.2847	0.0245	0.0638	31.29	0.769322 (18)	0.534852 (18)	0.437167(18)
FLD18	19,853.38	5,974	0.4398	0.0337	0.0179	25.12	0.913731 (12)	0.767148 (4)	0.673624(4)
FLD19	20,355	17,402	0.4421	0.0856	0.0217	30.02	0.923829 (10)	0.790033 (2)	0.660799(5)
Optimal Common Set of Weights									
							0.0000531527	0.01000775	0.0087323
							0.0000095774	0.00154359	0.0000096
							1.7985943288	326.66210793	265.1125134
							1.9470404984	414.56952964	1.9470405
							1.9700551615	96.94981070	1.9700552
							0.0049603175	0.00496032	0.0049603

Table 2. Correlation test of ranking models.

		Spearman's Rank Correlation		
		Wang and Jiang [11] Model (1)	**Özsoy et al. [1] Model (2)**	**Proposed Model**
Wang and Jiang [11]	Correlation	1	0.81614751	0.57393599
Model (1)	p-value		(2.03464496586902e-05)	(0.0101817517746076)
Özsoy et al. [1]	Correlation		1	0.78245614
Model (2)	p-value			(7.51884977459034e-05)
Proposed model	Correlation			1
	p-value			

5 | Conclusion

This paper presented a straightforward MILP model designed to identify the most efficient DMU using a common set of weights. The model simplifies the evaluation process by reducing the number of constraints while still offering strong discriminatory power. Through testing on well-known case studies, the model proved both effective and practical. Overall, the approach shows promise as a useful tool for efficiency analysis. Looking ahead, future research could explore how different choices for the parameter M affect the results and how the model can be adapted to handle negative data.

Conflict of Interest

The authors declare no conflict of interest.

Data Availability

All data are included in the text.

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