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Modeling Uncertainty in Logistics: Optimizing Fuzzy Transportation under Interval-Valued Trapezoidal Fuzzy Numbers

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Abstract

This study addresses the transportation problem under imprecise conditions by employing Interval-Valued Trapezoidal Fuzzy Numbers (IVTrFNs) to model uncertain parameters including transportation costs, supplies, and demands. The primary contribution of this research is the development of a novel optimization framework based on a weighted linear ranking function, complemented by a thorough theoretical analysis of its properties. This function effectively transforms IVTrFN parameters into crisp values through a parameter z, which allows the decision-maker to incorporate their strategic outlook by weighting the lower and upper bounds of the fuzzy intervals. The proposed method significantly streamlines the solution process, reducing the computational burden commonly associated with complex fuzzy arithmetic operations. The efficacy of the approach is validated through a comprehensive numerical example involving two supply sources and three demand destinations. The computational results reveal a compelling insight: as the parameter z increases from 0 to 1, reflecting a transition from a pessimistic to an optimistic outlook, the total transportation cost decreases by approximately 2.59 times. This finding carries substantial implications for logistics management, demonstrating that strategic orientation toward uncertainty can dramatically influence operational costs. The proposed framework equips managers with a practical, theoretically grounded decision-support tool for designing optimal transportation strategies under uncertainty.

Keywords: Interval-valued trapezoidal fuzzy number, Ranking method, Transportation problem, Fuzzy optimization, Risk preference.

1 | Introduction

The transportation problem represents one of the most fundamental and widely applied models in operations research and supply chain management. First formulated by Hitchcock in 1941 [1], the classical transportation problem seeks to determine the optimal shipment plan for distributing goods from multiple supply origins to

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various demand destinations, with the objective of minimizing total transportation cost while satisfying supply and demand constraints. Despite its apparent simplicity, the transportation problem has evolved into a rich research domain, spawning numerous extensions and generalizations that address increasingly complex real-world scenarios.

In classical formulations, the transportation problem assumes that all parameters, including unit transportation costs, available supplies, and required demands, are precisely known and deterministic. However, in practical logistics and supply chain operations, such certainty is rarely attainable. Transportation costs fluctuate due to fuel price volatility, changing market conditions, route congestion, and seasonal variations. Similarly, supply availability may be affected by production disruptions, while customer demand often exhibits inherent unpredictability. These uncertainties pose significant challenges for decision-makers who must formulate robust and cost-effective transportation strategies.

Fuzzy set theory provides a powerful mathematical framework for modeling and reasoning under uncertainty and imprecision. Unlike probabilistic approaches that require historical data and statistical assumptions, fuzzy set theory accommodates linguistic and subjective information, making it particularly suitable for situations where precise numerical data are unavailable or costly to obtain. In the context of transportation problems, fuzzy set theory enables decision-makers to represent vague or incomplete information about costs, supplies, and demands using fuzzy numbers that capture the inherent imprecision of these parameters.

Since the pioneering work of Liu and Kao [2], who first formulated the fuzzy transportation problem using the extension principle, numerous researchers have explored various fuzzy approaches to transportation optimization. These developments have followed several distinct trajectories, each addressing different aspects of uncertainty modeling.

Chandran and Kandaswamy [3] proposed an effective method for solving transportation problems in which costs, supplies, and demands are represented as fuzzy numbers. Their approach demonstrated that fuzzy formulations can yield more realistic and practical solutions compared to deterministic models. Saini et al. [4] introduced a ranking method specifically designed for unbalanced fuzzy transportation problems using generalized triangular and trapezoidal fuzzy numbers, addressing the important practical case where total supply does not equal total demand.

Baykasoglu and Subulan [5] developed a direct solution methodology based on constrained fuzzy arithmetic operations combined with metaheuristic techniques. Their approach provided a comprehensive framework for fully fuzzy transportation problems where all parameters are expressed as fuzzy numbers. The integration of constrained fuzzy arithmetic with metaheuristic optimization represented a significant step forward in handling the computational complexity inherent in fuzzy transportation problems.

In the domain of interval-valued fuzzy number modeling, Gupta and Kumar [6] investigated multi-objective transportation problems with fuzzy parameters, extending the applicability of fuzzy transportation models to scenarios involving conflicting objectives. Ebrahimnejad [7] specifically focused on Interval-Valued Trapezoidal Fuzzy Numbers (IVTrFNs), proposing a fuzzy linear programming approach for solving transportation problems with such representations. More recently, Ebrahimnejad et al. [8] introduced a novel optimization structure for solving transportation problems with interval-valued triangular fuzzy parameters, demonstrating the continued evolution and refinement of fuzzy transportation methodologies.

Despite these significant advancements, the existing literature reveals a notable gap: limited attention has been devoted to solving transportation problems using IVTrFNs combined with theoretically robust ranking functions that offer both computational efficiency and meaningful managerial interpretation. While interval-valued fuzzy numbers provide greater flexibility in representing uncertainty by capturing both the lower and upper bounds of membership functions, many existing approaches either rely on computationally intensive defuzzification procedures or lack comprehensive theoretical foundations.

Furthermore, the relationship between decision-maker attitudes toward uncertainty and optimal transportation outcomes has received insufficient attention. In practice, managers' strategic orientations ranging from optimistic to pessimistic significantly influence their interpretation of fuzzy information and subsequent decision-making. An effective transportation optimization framework should accommodate these varying perspectives and provide insights into how different strategic outlooks affect operational costs. This research addresses the identified gaps through several key contributions:

I. Novel ranking function: We propose a weighted linear ranking function specifically designed for IVTrFNs that combines mathematical elegance with practical applicability. The function incorporates a parameter z that allows decision-makers to adjust their emphasis on the lower versus upper bounds of fuzzy intervals.

II. Theoretical development: We establish three fundamental theorems demonstrating the linearity, monotonicity with respect to fuzzy values, and monotonic behavior with respect to the parameter z. These theoretical properties provide a rigorous foundation for the practical application of the ranking function.

III. Strategic outlook integration: The parameter z enables systematic incorporation of the decision-maker's strategic outlook into the optimization process, allowing for sensitivity analysis of how strategic orientation affects optimal transportation plans.

IV. Managerial insights: Through a detailed numerical example, we demonstrate the substantial impact of strategic outlook on total transportation costs, providing actionable insights for logistics managers.

The remainder of this paper is organized as follows. Section 2 presents the fundamental concepts and definitions of interval-valued trapezoidal fuzzy numbers, including their arithmetic operations. Section3 formulates the interval-valued transportation problem mathematically, explaining the constraints and objective function in detail. Section 4 develops the proposed solution methodology, including the weighted linear ranking function, the three theoretical theorems with complete proofs, and a step-by-step solution procedure. Section 5 provides comprehensive numerical results, including data description, ranking computations, optimal solutions, and sensitivity analysis. Section 6 concludes the study, discusses limitations, and suggests directions for future research.

2 | Basic Concepts and Definitions

This section presents the fundamental mathematical concepts necessary for understanding the proposed fuzzy transportation model. We begin with the definition of trapezoidal fuzzy numbers, then extend to IVTrFNs, and finally discuss the arithmetic operations that will be employed throughout this study.

Before introducing IVTrFNs, it is instructive to recall the definition of a standard trapezoidal fuzzy number, which serves as the building block for the more general interval-valued case.

Definition 1. A trapezoidal fuzzy number $\widetilde{A}$ is defined as an ordered quadruple $\widetilde{A} = (a_1, a_2, a_3, a_4; \lambda)$, where $a_1 \le a_2 \le a_3 \le a_4$, and $\lambda \in (0,1]$ represents the maximum degree of membership. The membership function $\mu_{\widetilde{A}}: R \to [0, \lambda]$ is defined as follows:

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{(x - a_1)\lambda}{a_2 - a_1}, & a_1 \le x < a_2, \\ \lambda, & a_2 \le x \le a_3, \\ \dfrac{(a_4 - x)\lambda}{a_4 - a_3}, & a_3 < x \le a_4, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

The support of the fuzzy number is the interval $[a_1, a_4]$, while the core is the interval $[a_2, a_3]$. The parameter λ controls the height of the fuzzy number, and when $\lambda = 1$, the fuzzy number is said to be normalized. For computational convenience, many applications restrict attention to normalized fuzzy numbers; however, the generalized case $(\lambda < 1)$ is sometimes useful to model situations where the expert's confidence level is not maximal.

Remark 1. A trapezoidal fuzzy number reduces to a triangular fuzzy number when a_2, a_3. In this case, the membership function forms a triangle with its peak at the point a_2, a_3. Furthermore, when $a_1 = a_2 = a_3 = a_4$, the fuzzy number degenerates to a crisp real number.

Example 1. Consider the trapezoidal fuzzy number $\tilde{A} = (2, 5, 7, 10;\ 0.8)$. The membership function is:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0.8\,\dfrac{x-2}{3}, & 2 \le x < 5, \\ 0.8, & 5 \le x \le 7, \\ 0.8\,\dfrac{10-x}{3}, & 7 < x \le 10, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

This fuzzy number represents a quantity that is "approximately between 5 and 7" with a confidence level of 0.8, and the possible range extends from 2 to 10 with diminishing membership.

In many practical situations, the exact value of the membership degree λ is itself uncertain. To address this, interval-valued fuzzy numbers were introduced, where the membership degree is allowed to range over an interval $[\underline{\lambda}, \overline{\lambda}]$. This approach provides a more flexible representation of uncertainty, capturing both the spread of the data and the imprecision in the expert's assessment of membership.

Definition 2. Let $\tilde{G}^l$ and $\tilde{G}^r$ be two trapezoidal fuzzy numbers at levels $\underline{\lambda}_{\tilde{G}}$ and $\overline{\lambda}_{\tilde{G}}$ on R, respectively. An IVTrFN is represented as:

$$\tilde{G} = \langle \tilde{G}^l, \tilde{G}^r \rangle = \langle (\underline{g}_1, \underline{g}_2, \underline{g}_3, \underline{g}_4; \underline{\lambda}_{\tilde{G}}), (\overline{g}_1, \overline{g}_2, \overline{g}_3, \overline{g}_4; \overline{\lambda}_{\tilde{G}}) \rangle.$$

With the following lower and upper membership functions [8], [9]:

$$\mu_{\tilde{G}^l}(x) = \begin{cases} \dfrac{\left(x-\underline{g}_1\right)\underline{\lambda}_{\tilde{G}}}{\underline{g}_2-\underline{g}_1} & \underline{g}_1 \le x < \underline{g}_2, \\ \underline{\lambda}_{\tilde{G}} & \underline{g}_2 \le x < \underline{g}_3, \\ \dfrac{\left(\underline{g}_4-x\right)\underline{\lambda}_{\tilde{G}}}{\underline{g}_4-\underline{g}_3} & \underline{g}_3 \le x < \underline{g}_4, \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

And

$$\mu_{\tilde{G}^r}(x) = \begin{cases} \dfrac{(x-\overline{g}_1)\overline{\lambda}_{\tilde{G}}}{\overline{g}_2-\overline{g}_2} & \overline{g}_1 \le x < \overline{g}_2, \\ \overline{\lambda}_{\tilde{G}} & \overline{g}_2 \le x < \overline{g}_3, \\ \dfrac{(\overline{g}_4-x)\overline{\lambda}_{\tilde{G}}}{\overline{g}_4-\overline{g}_3} & \overline{g}_3 \le x < \overline{g}_4, \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

In this representation, the membership degree is an interval $[\underline{\lambda}_{\tilde{G}}, \overline{\lambda}_{\tilde{G}}]$ rather than a single number. The lower membership function $\mu_{\tilde{G}^l}$ defines the guaranteed or conservative membership level, while the upper membership function $\mu_{\tilde{G}^r}$ defines the optimistic membership level. This dual structure allows the model to capture both the inherent vagueness of the data and the uncertainty in the expert's judgment about that vagueness.

Remark 2. When $\underline{\lambda}_{\tilde{G}} = \overline{\lambda}_{\tilde{G}}$, the interval-valued trapezoidal fuzzy number reduces to a standard trapezoidal fuzzy number. Therefore, *Definition 2* generalizes the classical concept.

Example 2. Le$\widetilde{G} = \langle(2, 4, 6, 8; 0.6), (1, 4, 6, 9; 1.0)\rangle$t. The lower membership function has a height of 0.6, while the upper membership function has a height of 1.0. The support of the lower fuzzy number is [3], [9], while the support of the upper fuzzy number is [4], [5], reflecting the wider range of possible values at the upper membership level.

To perform mathematical operations with IVTrFNs, it is necessary to define appropriate arithmetic operations. These operations extend the standard fuzzy arithmetic to the interval-valued setting, preserving the interval nature of the membership degrees.

Definition 3. Let $\widetilde{G} = \langle \widetilde{G}^l, \widetilde{G}^r \rangle = \langle \left(\underline{g}_1, \underline{g}_2, \underline{g}_3, \underline{g}_4; \underline{\lambda}\right), (\overline{g}_1, \overline{g}_2, \overline{g}_3, \overline{g}_4; \overline{\lambda}) \rangle$ and $\widetilde{H} = \langle \widetilde{H}^l, \widetilde{H}^r \rangle = \langle (\underline{h}_1, \underline{h}_2, \underline{h}_3, \underline{h}_4; \underline{\lambda}), (\overline{h}_1, \overline{h}_2, \overline{h}_3, \overline{h}_4; \overline{\lambda}) \rangle$ be two IVTrFNs at levels $\underline{\lambda}$ and $\overline{\lambda}$ on R, and let δ be a non-negative scalar. The following arithmetic operations are defined [8], [9]:

I. $\widetilde{G} + \widetilde{H} = \langle (\underline{g}_1 + \underline{h}_1, \underline{g}_2 + \underline{h}_2, \underline{g}_3 + \underline{h}_3, \underline{g}_4 + \underline{h}_4; \underline{\lambda}), (\overline{g}_1 + \overline{h}_1, \overline{g}_2 + \overline{h}_2, \overline{g}_3 + \overline{h}_3, \overline{g}_4 + \overline{h}_4; \overline{\lambda}) \rangle$.

II. $\delta\widetilde{G} = \langle (\delta\underline{g}_1, \delta\underline{g}_2, \delta\underline{g}_3, \delta\underline{g}_4; \underline{\lambda}), (\delta\overline{g}_1, \delta\overline{g}_2, \delta\overline{g}_3, \delta\overline{g}_4; \overline{\lambda}) \rangle$.

Remark 3. The addition operation is commutative and associative. Scalar multiplication distributes over addition: $\delta(\widetilde{G} + \widetilde{H}) = \delta\,\widetilde{G} + \delta\,\widetilde{H}$. These properties are essential for the linearity of the subsequent ranking function.

Example 3. Let $\widetilde{G} = \langle (2,4,6,8; 0.6), (1,4,6,9; 1.0) \rangle$ and $\widetilde{H} = \langle (3, 5, 7, 9;\ 0.7), (2, 5, 7, 10;\ 1.0) \rangle$. Then:

$$\widetilde{G} + \widetilde{H} = \langle (5, 9, 13, 17;\ 0.6), (3, 9, 13, 19;\ 1.0) \rangle.$$

and for $\delta = 2$:

$$2\widetilde{G} = \langle (4, 8, 12, 16;\ 0.6), (2, 8, 12, 18;\ 1.0) \rangle.$$

A critical issue in fuzzy optimization is the need to compare and rank fuzzy numbers. Unlike crisp numbers, fuzzy numbers do not possess a natural total order. Various ranking methods have been proposed in the literature, each with its own advantages and limitations. The choice of ranking method significantly influences the resulting optimization solution, as it determines how fuzzy objectives and constraints are converted into crisp equivalents.

Many existing ranking methods rely on the centroid or expected value of the fuzzy number, while others use various indices of the membership function. However, for interval-valued trapezoidal fuzzy numbers, the ranking problem becomes more complex because the membership degree itself is an interval. The ranking method introduced in Section4 addresses this challenge by providing a weighted linear function that accounts for both the fuzzy values and the interval-valued membership degrees.

3 | Mathematical Model of the Interval-Valued Transportation Problem

This section presents the mathematical formulation of the transportation problem in an interval-valued fuzzy environment. We begin by introducing the basic structure of the classical transportation problem, then extend it to accommodate interval-valued trapezoidal fuzzy parameters, and finally discuss the feasibility conditions and economic interpretation of the model.

3.1 | Classical Transportation Problem

The classical transportation problem, first formulated by Hitchcock in 1941 [1] and independently by Koopmans in 1949 [10], seeks to determine the optimal distribution of a homogeneous commodity from a set of supply origins to a set of demand destinations. The objective is to minimize total transportation cost while satisfying all supply and demand constraints.

In the deterministic case, the problem can be stated as follows. Suppose there are m sources (supply points) $(S_1, S_2, \dots, S_m)$ with available supplies $(a_1, a_2, \dots, a_m)$, and n destinations (demand points) $(D_1, D_2, \dots, D_n)$ with required demands $(b_1, b_2, \dots, b_n)$. Let (c_{ij}) denote the unit transportation cost from source i to destination j, and let (y_{ij}) represent the quantity shipped from source i to destination j. The problem is formulated as:

$$
\begin{aligned}
&\text{Minimize } O = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} y_{ij}.\\
&\text{s.t.}\\
&\sum_{j=1}^{n} y_{ij} = a_i, \quad i = 1,2,\dots,m.\\
&\sum_{i=1}^{m} y_{ij} = b_j, \quad j = 1,2,\dots,n.\\
&y_{ij} \geq 0, \qquad \forall i, j.
\end{aligned}
\tag{5}
$$

The first set of constraints ensures that the total quantity shipped from each source equals its available supply. The second set ensures that the total quantity received at each destination equals its required demand. The non-negativity constraints reflect the physical impossibility of negative shipments. A necessary and sufficient condition for feasibility is $(\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j)$, known as the balance condition. When this condition holds, the problem is said to be balanced; otherwise, it is unbalanced, requiring the introduction of dummy sources or destinations.

3.2 | Fuzzy Transportation Problem with Interval-Valued Parameters

In the real world, the exact values of transportation costs, supplies, and demands are often imprecise due to various sources of uncertainty such as fluctuating fuel prices, variable production capacities, unpredictable customer demand, and incomplete information about market conditions. Traditional deterministic models fail to capture this uncertainty adequately, potentially leading to suboptimal or even infeasible solutions in practice.

To address this limitation, we extend the classical transportation problem to an interval-valued fuzzy environment. In our formulation, the unit transportation cost $\tilde{c}_{ij}$, the available supply $\tilde{a}_i$, and the required demand $\tilde{b}_j$ are all represented as IVTrFNs. This representation offers several advantages:

I. Flexibility: IVTrFNs capture both the spread of possible values and the imprecision in membership degrees, providing a richer representation of uncertainty compared to standard fuzzy numbers.

II. Robustness: By incorporating interval-valued membership levels, the model accommodates varying degrees of expert confidence in the data.

III. Practicality: The interval structure reflects the natural way that decision-makers often express their knowledge.

Definition 4. Let $(\tilde{c}_{ij} \in \mathcal{F}(R))$ denote the interval-valued trapezoidal fuzzy cost for transporting one unit from source i to destination j, $(\tilde{a}_i \in \mathcal{F}(R))$ denote the fuzzy supply available at source i, and also, $(\tilde{b}_j \in \mathcal{F}(R))$ denote the fuzzy demand required at destination j. The interval-valued fuzzy transportation problem is formulated as:

$$\text{Minimize } O = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij}\, y_{ij},$$

$$\text{s.t.}$$

$$\sum_{j=1}^{n} y_{ij} = \tilde{a}_i, \qquad i = 1,2,\dots,m, \tag{6}$$

$$\sum_{i=1}^{m} y_{ij} = \tilde{b}_j, \qquad j = 1,2,\dots,n,$$

$$y_{ij} \geq 0, \qquad \forall i, j.$$

In this formulation, y_{ij} are crisp decision variables representing the quantity shipped from source i to destination j. The objective function involves fuzzy costs $\tilde{c}_{ij}$ multiplied by the crisp variables, resulting in a fuzzy total cost. The constraints require that the sum of shipments from each source equals the fuzzy supply, and the sum of shipments to each destination equals the fuzzy demand.

Remark 1. The decision variables y_{ij} are assumed to be crisp because, in practice, shipment quantities are determined precisely by the decision-maker. The uncertainty lies solely in the parameters (costs, supplies, demands), not in the decisions themselves.

Remark 2. For the transportation problem to be feasible in a fuzzy sense, the total fuzzy supply must equal the total fuzzy demand. That is:

$$\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j. \tag{7}$$

When this condition holds, the problem is said to be balanced in the fuzzy sense. The equality of fuzzy numbers requires equality of both the lower and upper membership functions. In practice, this condition may be difficult to achieve exactly, and approximations or transformations may be necessary.

To facilitate understanding of the model, we now provide an interpretation of each constraint in the context of the transportation problem.

The supply constraints $\sum_{j=1}^{n} y_{ij} = \tilde{a}_i$ require that the total quantity shipped from source i to all destinations equals the fuzzy supply available at source i. This constraint reflects the physical limitation that a source cannot ship more than its available supply. The use of fuzzy equality acknowledges that the supply quantity is not precisely known; it may vary within a range described by the IVTrFN $\tilde{a}_i$.

In practical terms, consider a warehouse where the exact inventory level is uncertain due to ongoing production, quality control issues, or measurement errors. The IVTrFN $\tilde{a}_i$ captures this uncertainty, and the constraint ensures that the shipping plan respects this uncertain inventory.

The demand constraints $\sum_{i=1}^{m} y_{ij} = \tilde{b}_j$ require that the total quantity received at destination j from all sources equals the fuzzy demand at destination j. This constraint ensures that customer requirements are met. The use of fuzzy demand acknowledges that customer orders may be subject to variability or that the exact demand is not known with certainty. For example, a retailer may have an uncertain demand for a product due to seasonal fluctuations, promotional activities, or market uncertainty. The IVTrFN $\tilde{b}_j$ captures this uncertainty, and the constraint ensures that the shipping plan satisfies the uncertain demand.

The constraints $y_{ij} \geq 0$ are standard in transportation problems, ensuring that shipments are in the correct direction (from sources to destinations) and that reverse shipments are not allowed. This constraint is a fundamental physical requirement in logistics operations.

As noted in *Remark 2*, a necessary condition for the existence of a feasible solution is that the total fuzzy supply equals the total fuzzy demand. For IVTrFNs, this condition takes the following form.

Let $\tilde{a}_j = \langle (\underline{a}_{i1}, \underline{a}_{i2}, \underline{a}_{i3}, \underline{a}_{i4}; \underline{\lambda}_i), (\bar{a}_{i1}, \bar{a}_{i2}, \bar{a}_{i3}, \bar{a}_{i4}; \bar{\lambda}_i) \rangle$ and $\tilde{b}_j = \left\langle \left(\underline{b}_{j1}, \underline{b}_{j2}, \underline{b}_{j3}, \underline{b}_{j4}; \underline{\mu}_j \right), \left(\bar{b}_{j1}, \bar{b}_{j2}, \bar{b}_{j3}, \bar{b}_{j4}; \bar{\mu}_j \right) \right\rangle$. Then $\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$ if and only if:

$$\sum_{i=1}^{m} \underline{a}_{ik} = \sum_{j=1}^{n} \underline{b}_{jk}, \quad k = 1,2,3,4,$$

$$\sum_{i=1}^{m} \bar{a}_{ik} = \sum_{j=1}^{n} \bar{b}_{jk}, \quad k = 1,2,3,4, \tag{8}$$

$$\underline{\lambda}_i = \underline{\mu}_j, \quad \bar{\lambda}_i = \bar{\mu}_j, \quad \forall i, j.$$

In practice, these conditions may not hold exactly. In such cases, the problem is said to be unbalanced in the fuzzy sense. For unbalanced fuzzy transportation problems, several approaches can be employed:

I. Introduction of dummy sources or destinations: Similar to classical unbalanced problems, dummy variables can be introduced to absorb the excess supply or demand. The costs associated with dummy variables are typically zero or reflect penalty costs.

II. Relaxation of equality constraints: The equality constraints can be relaxed to inequality constraints, allowing for excess supply or shortage. This approach is suitable when overproduction or unmet demand is acceptable.

III. Defuzzification first: The fuzzy parameters can be converted to crisp values using an appropriate ranking method, and then the classical unbalanced problem can be solved.

In this study, we assume that the problem is balanced, which is consistent with the numerical example presented in Section 5.

4 | Proposed Solution Methodology

This section presents the core methodological contribution of this research: a weighted linear ranking function for IVTrFNs, along with its theoretical properties and a systematic solution procedure for the fuzzy transportation problem. The proposed approach bridges the gap between fuzzy parameter representation and crisp optimization by providing a theoretically grounded transformation mechanism that preserves meaningful information while enabling efficient computation.

4.1 | Weighted Linear Ranking Function

A fundamental challenge in fuzzy optimization is the need to compare and rank fuzzy numbers. Unlike crisp numbers, fuzzy numbers do not possess a natural total order, and various ranking methods have been proposed in the literature, each with distinct advantages and limitations. For IVTrFNs, the ranking problem becomes more complex due to the dual structure of lower and upper membership functions.

Let $\tilde{G} = \langle \tilde{G}^l, \tilde{G}^r \rangle = \left\langle \left(\underline{g}_1, \underline{g}_2, \underline{g}_3, \underline{g}_4; \underline{\lambda} \right), \left(\bar{g}_1, \bar{g}_2, \bar{g}_3, \bar{g}_4; \bar{\lambda} \right) \right\rangle$ be an interval-valued trapezoidal fuzzy number. Based on the ranking methodology proposed by De et al. [11], we define a weighted linear ranking function that captures both the position and spread of the fuzzy number while incorporating the decision-maker's strategic outlook through a single parameter $z \in [0,1]$.

The ranking function is constructed in two stages. First, we compute the ranking values for the lower and upper membership functions separately:

I. Lower ranking value: $S_z(\tilde{G}^l) = \frac{1}{2}\left\{ z\underline{\lambda}\left(\underline{g}_1 + \underline{g}_2 \right) + (1 - z)\underline{\lambda}\left(\underline{g}_3 + \underline{g}_4 \right) \right\}$,

II. Upper ranking value: $S_z(\tilde{G}^r) = \frac{1}{2}\left\{ z\bar{\lambda}(\bar{g}_1 + \bar{g}_2) + (1 - z)\bar{\lambda}(\bar{g}_3 + \bar{g}_4) \right\}$, where $z \in [0,1]$ is a parameter chosen by the decision-maker. The parameter z controls the relative weight assigned to the left-hand (lower) versus right-hand (upper) portions of the fuzzy number's support. Specifically:

I. When $z = 1$, the ranking value is based entirely on the left endpoints (g_1, g_2), emphasizing the lower (more optimistic) values of the fuzzy interval.

II. When $z = 0$, the ranking value is based entirely on the right endpoints (g_3, g_4), emphasizing the upper (more pessimistic) values.

III. Intermediate values of z provide a weighted average of both extremes.

The overall ranking value of the interval-valued trapezoidal fuzzy number is then computed as the average of the lower and upper ranking values:

$$S_z(\widetilde{G})=\frac{1}{4}\left\{z\left(\underline{\lambda}\left(\underline{g}_1+\underline{g}_2\right)+\overline{\lambda}\left(\overline{g}_1+\overline{g}_2\right)\right)+(1-z)\left(\underline{\lambda}\left(\underline{g}_3+\underline{g}_4\right)+\overline{\lambda}\left(\overline{g}_3+\overline{g}_4\right)\right)\right\}. \quad (9)$$

Remark 1. The ranking function defined in *Eq. (9)* is a scalar-valued function that maps each interval-valued trapezoidal fuzzy number to a real number. This transformation enables the conversion of fuzzy parameters into crisp values, facilitating the use of standard linear programming techniques.

Example 1. Consider the interval-valued trapezoidal fuzzy number $\widetilde{G} = \langle(2,4,6,8; 0.6), (1,4,6,9; 1.0)\rangle$ For $z = 0.5$

$$S_{0.5}(\widetilde{G}^l) = \frac{1}{2}\{0.5 \times 0.6 \times (2+4) + 0.5 \times 0.6 \times (6+8)\} = 3.0,$$

and

$$S_{0.5}(\widetilde{G}^r) = \frac{1}{2}\{0.5 \times 1.0 \times (1+4) + 0.5 \times 1.0 \times (6+9)\} = \frac{1}{2} \times (2.5 + 7.5) = 5.0,$$

$$S_{0.5}(\widetilde{G}) = \frac{3.0 + 5.0}{2} = 4.$$

For $z = 1$ (optimistic outlook):

$$S_1(\widetilde{G}^l) = \frac{1}{2}\{1 \times 0.6 \times (2+4) + 0\} = 1.8 \quad \text{and} \quad S_1(\widetilde{G}^r) = \frac{1}{2}\{1 \times 1.0 \times (1+4) + 0\} = 2.5.$$

$$S_1(\widetilde{G}) = \frac{1.8 + 2.5}{2} = 2.15.$$

For $z = 0$ (pessimistic outlook):

$$S_0(\widetilde{G}^l) = \frac{1}{2}\{0 + 1 \times 0{,}6 \times (6+8)\} = 4.2 \quad S_0(\widetilde{G}^r) = \frac{1}{2}\{0 + 1 \times 1.0 \times (6+9)\} = 7.5.$$

$$S_0(\widetilde{G}) = \frac{4.2 + 7.5}{2} = 5.85.$$

This example illustrates that as z increases from 0 to 1, the ranking value decreases (5.85 → 4.0 → 2.15). This behavior is formally established in *Theorem 3*.

4.2 | Theoretical Properties

For a ranking function to be considered valid and useful in optimization contexts, it must satisfy certain fundamental properties. The proposed ranking function $S_z(0)$ exhibits three important properties, which are formally stated and proved below.

Theorem 1. The ranking function $S_z(0)$ is linear. That is, for any two IVTrFNs $\tilde{G}$ and $\tilde{H}$, and any non-negative scalar $\delta \geq 0$:

$$S_z(\delta\tilde{G}+\tilde{H})=\delta S_z(\tilde{G})+S_z(\tilde{H}).$$

Proof: Let $\tilde{G}=\left\langle\left(\underline{g}_1,\underline{g}_2,\underline{g}_3,\underline{g}_4;\underline{\lambda}\right),\left(\overline{g}_1,\overline{g}_2,\overline{g}_3,\overline{g}_4;\overline{\lambda}\right)\right\rangle$ and $\tilde{H}=\langle(\underline{h}_1,\underline{h}_2,\underline{h}_3,\underline{h}_4;\underline{\lambda}),(\overline{h}_1,\overline{h}_2,\overline{h}_3,\overline{h}_4;\overline{\lambda})\rangle$ be two IVTrFNs at the same levels $\underline{\lambda}$ and $\overline{\lambda}$. Using the arithmetic operations, we have:

$$\delta\tilde{G}+\tilde{H}=\left\langle\left(\delta\underline{g}_1+\underline{h}_1,\delta\underline{g}_2+\underline{h}_2,\delta\underline{g}_3+\underline{h}_3,\delta\underline{g}_4+\underline{h}_4;\underline{\lambda}\right),\left(\delta\overline{g}_1+\overline{h}_1,\delta\overline{g}_2+\overline{h}_2,\delta\overline{g}_3+\overline{h}_3,\delta\overline{g}_4\right.\right.$$
$$\left.+\overline{h}_4;\overline{\lambda}\right)$$

Applying the ranking function $S_z(.)$:

$$S_z(\delta\tilde{G}+\tilde{H})=\frac{1}{4}(z\left(\underline{\lambda}\left(\delta\underline{g}_1+\underline{h}_1+\delta\underline{g}_2+\underline{h}_2\right)+\overline{\lambda}(\delta\overline{g}_1+\overline{h}_1+\delta\overline{g}_2+\overline{h}_2)\right)$$
$$+(1-z)\left(\underline{\lambda}\left(\delta\underline{g}_3+\underline{h}_3+\delta\underline{g}_4+\underline{h}_4\right)+\overline{\lambda}(\delta\overline{g}_3+\overline{h}_3+\delta\overline{g}_4+\overline{h}_4)\right)$$

Rearranging terms:

$$S_z(\delta\tilde{G}+\tilde{H})=$$
$$\frac{1}{4}\left(z\left(\underline{\lambda}\left(\delta\underline{g}_1+\delta\underline{g}_2\right)+\overline{\lambda}(\delta\overline{g}_1+\delta\overline{g}_2)\right)+(1-z)\left(\underline{\lambda}\left(\delta\underline{g}_3+\delta\underline{g}_4\right)+\overline{\lambda}(\delta\overline{g}_3+\delta\overline{g}_4)\right)\right)$$
$$+\frac{1}{4}\left(z\left(\underline{\lambda}(\underline{h}_1+\underline{h}_2)+\overline{\lambda}(\overline{h}_1+\overline{h}_2)\right)+(1-z)\left(\underline{\lambda}(\underline{h}_3+\underline{h}_4)+\overline{\lambda}(\overline{h}_3+\overline{h}_4)\right)\right).$$

Factoring δ:

$$S_z(\delta\,\tilde{G}+\tilde{H})=\frac{\delta}{4}\left(z\left(\underline{\lambda}\left(\underline{g}_1+\underline{g}_2\right)+\overline{\lambda}(\overline{g}_1+\overline{g}_2)\right)\right.$$
$$\left.+(1-z)\left(\underline{\lambda}\left(\underline{g}_3+\underline{g}_4\right)+\overline{\lambda}(\overline{g}_3+\overline{g}_4)\right)\right)$$
$$+\frac{1}{4}\left(z\left(\underline{\lambda}(\underline{h}_1+\underline{h}_2)+\overline{\lambda}(\overline{h}_1+\overline{h}_2)\right)+(1-z)\left(\underline{\lambda}(\underline{h}_3+\underline{h}_4)+\overline{\lambda}(\overline{h}_3+\overline{h}_4)\right)\right).$$

Recognizing the definitions of $S_z(\tilde{G})$ and $S_z(\tilde{H})$:

$$S_z(\delta\tilde{G}+\tilde{H})=\delta S_z(\tilde{G})+S_z(\tilde{H}).$$

Thus, the linearity property is established. ■

Remark 2. The linearity property is essential for the application of the ranking function in linear programming contexts. It ensures that the ranking of a linear combination of fuzzy numbers is equal to the same linear combination of their individual rankings, preserving the linear structure of the optimization model.

Theorem 2. The ranking function $S_z(\tilde{G})$ is monotonically increasing with respect to the trapezoidal values of $\tilde{G}=\langle\tilde{G}^l,\tilde{G}^r\rangle$. That is, if all components of $\tilde{G}$ increase component-wise, then $S_z(\tilde{G})$ also increases.

Proof: Consider two IVTrFNs $\tilde{G}$ and $\tilde{G}'$ such that $\tilde{G}\leq\tilde{G}'$ component-wise. That is:

$$\underline{g}_i \leq \underline{g_i'},\ \overline{g}_i \leq \overline{g_i'},\ i = 1,2,3,4,$$

and

$$\underline{\lambda} \leq \underline{\lambda'}, \overline{\lambda} \leq \overline{\lambda'}$$

Since all terms in $S_z(.)$ are non-negative (due to $z \in [0,1]$, $\underline{\lambda} > 0$, $\overline{\lambda} > 0$, and $\underline{g}_1 \leq \underline{g}_2 \leq \underline{g}_3 \leq \underline{g}_4$, the function $S_z(.)$ is a linear combination of these components with non-negative coefficients. Therefore, any increase in the components results in a corresponding increase in the value of $S_z(.)$. Formally, the partial derivatives with respect to each component are non-negative:

$$\frac{\partial S_z}{\partial \underline{g}_i} \geq 0, \quad \frac{\partial S_z}{\partial \overline{g}_i} \geq 0, \quad i = 1,2,3,4.$$

and

$$\frac{\partial S_z}{\partial \underline{\lambda}} \geq 0, \quad \frac{\partial S_z}{\partial \overline{\lambda}} \geq 0.$$

Hence, $S_z(0)$ is monotonically increasing with respect to all its arguments. ■

Remark 3. This monotonicity property ensures that the ranking function preserves the natural ordering of fuzzy numbers: larger fuzzy values (in the sense of component-wise ordering) receive larger ranking scores. This property is an intuitive and desirable property for any ranking method.

Theorem 3. The ranking function $S_z(\widetilde{G})$ is monotonically decreasing with respect to the parameter z.

Proof. Taking the derivative of $S_z(\widetilde{G})$ with respect to z:

$$\frac{\partial S_z(\widetilde{G})}{\partial z} = \frac{1}{4}\left(\underline{\lambda}\left(\underline{g}_1 + \underline{g}_2\right) + \overline{\lambda}(\overline{g}_1 + \overline{g}_2) - \underline{\lambda}\left(\underline{g}_3 + \underline{g}_4\right) - \overline{\lambda}(\overline{g}_3 + \overline{g}_4)\right)$$

Since the fuzzy number is trapezoidal, we have $\underline{g}_1 \leq \underline{g}_2 \leq \underline{g}_3 \leq \underline{g}_4$ and $\overline{g}_1 \leq \overline{g}_2 \leq \overline{g}_3 \leq \overline{g}_4$. Consequently:

$$\underline{g}_1 + \underline{g}_2 \leq \underline{g}_3 + \underline{g}_4, \quad \overline{g}_1 + \overline{g}_2 \leq \overline{g}_3 + \overline{g}_4.$$

Therefore:

$$\underline{\lambda}(\underline{g}_1 + \underline{g}_2) + \overline{\lambda}\left(\overline{g}_1 + \overline{g}_2\right) \leq \underline{\lambda}(\underline{g}_3 + \underline{g}_4) + \overline{\lambda}\left(\overline{g}_3 + \overline{g}_4\right).$$

Thus:

$$\frac{\partial S_z(\widetilde{G})}{\partial z} \leq 0.$$

The derivative is strictly negative unless $\underline{g}_1 + \underline{g}_2 = \underline{g}_3 + \underline{g}_4$ and $\overline{g}_1 + \overline{g}_2 = \overline{g}_3 + \overline{g}_4$, which would correspond to a degenerate (crisp) case.

This decreasing behavior occurs because a higher z assigns more weight to the lower bounds of the fuzzy interval, which typically represent less costly or more certain outcomes. Conversely, a lower z assigns more weight to the upper bounds, reflecting a more pessimistic assessment. ■

Remark 4. The monotonicity with respect to z establishes a clear relationship between the decision-maker's strategic outlook and the resulting ranking values. This property is central to the sensitivity analysis presented in the numerical example.

4.3|Interpretation of the Parameter z

The parameter $z \in [0,1]$ plays a dual role in the proposed methodology:

I. Mathematical role: It serves as a weight that determines the relative emphasis placed on the left (lower) versus right (upper) portions of the fuzzy number's support in the ranking calculation.

II. Decision-making role: It reflects the strategic outlook of the decision-maker:

- $z = 1$*: The decision-maker adopts an optimistic outlook, focusing on the lower (more favorable) values.*
- $z = 0$*: The decision-maker adopts a pessimistic outlook, focusing on the upper (less favorable) values.*
- $0 < z < 1$*: The decision-maker adopts a balanced outlook, reflecting a compromise between optimism and pessimism.*

It is important to emphasize that the interpretation of z in terms of "optimistic" versus "pessimistic" outlook depends on the specific context of the problem. In the transportation problem, where the objective is cost minimization:

I. A higher z (optimistic) leads to lower cost estimates and, consequently, lower total transportation costs.

II. A lower z (pessimistic) leads to higher cost estimates and, consequently, higher total transportation costs.

This relationship is confirmed by the numerical results presented in Section 5, where increasing z from 0 to 1 results in a substantial decrease in total cost.

4.4|Solution Procedure

The proposed methodology for solving the interval-valued fuzzy transportation problem consists of the following systematic steps:

Step 1 (parameter conversion). For each interval-valued trapezoidal fuzzy parameter in the problem (costs $\tilde{c}_{ij}$, supplies $\tilde{a}_i$, and demands $\tilde{b}_j$), compute its crisp equivalent using the weighted linear ranking function $S_z(.)$ as defined in *Eq. (9)*, based on the decision-maker's chosen value of z.

$$c_{ij} = S_z(\tilde{c}_{ij}), \quad b_j = S_z(\tilde{b}_j).$$

Step 2 (crisp model construction). Substitute the crisp values obtained in *Step 1* into the transportation model, resulting in a standard deterministic linear programming problem.

Step 3 (optimization). Solve the crisp linear programming model using an appropriate optimization solver. In this study, we employ MATLAB 2018 with the built-in linear programming solver `linprog` to obtain the optimal shipment quantities y_{ij}^* and the minimum total cost O^*.

Step 4 (sensitivity analysis). Repeat *Steps 1-3* for different values of $z \in [0,1]$ to analyze how changes in the decision-maker's strategic outlook affect the optimal solution. This analysis provides valuable insights into the trade-offs between cost and outlook preference.

5|Numerical Results

This section presents a comprehensive numerical example to demonstrate the applicability and effectiveness of the proposed methodology. The example involves a transportation problem with two supply sources and three demand destinations, where all parameters transportation costs, supplies, and demands are represented as IVTrFNs. We systematically apply the proposed ranking function for various values of the parameter z, solve the resulting crisp linear programming models, and analyze the sensitivity of the optimal solution to changes in the decision-maker's strategic outlook.

5.1 | Problem Description

Consider a logistics company that operates with two supply sources s_1, s_2 and three demand destinations D_1, D_2, D_3. The company needs to determine the optimal shipment plan for distributing a homogeneous product from the sources to the destinations. However, the exact values of the transportation costs, available supplies, and required demands are not known with certainty due to various sources of uncertainty such as fluctuating fuel prices, variable production capacities, and unpredictable customer demand.

Following the proposed methodology, we represent all uncertain parameters as IVTrFNs. The data, structured according to the mathematical model presented in *Eq. (6)*, are as follows [5]:

Transportation Costs (IVTrFNs):

$$\tilde{c}_{11} = \langle [10,12,13,14; 0.5], [9,12,13,15; 1] \rangle.$$

$$\tilde{c}_{12} = \langle [8,10,11,12; 0.5], [7,10,11,14; 1] \rangle.$$

$$\tilde{c}_{13} = \langle [10,12,13,14; 0.5], [9,12,13,16; 1] \rangle.$$

$$\tilde{c}_{21} = \langle [2,3,7,8; 0.5], [1,3,7,9; 1] \rangle.$$

$$\tilde{c}_{22} = \langle [4,8,10,11; 0.5], [3,8,10,12; 1] \rangle.$$

$$\tilde{c}_{23} = \langle [5,7,8,9; 0.5], [4,7,8,10; 1] \rangle.$$

Here, for example, $\tilde{c}_{11}$ represents the fuzzy cost of transporting one unit from source s_1 to destination D_1. The lower membership function [10,12,13,14;0.5] indicates a conservative estimate with confidence level 0.5, while the upper membership function [9,12,13,15;1] represents an optimistic estimate with full confidence.

Supplies (IVTrFNs):

$$\tilde{a}_1 = \langle [2000,3000,4000,4500; 0.5]\ [1500,3000,4000,5000; 1] \rangle,$$

$$\tilde{a}_2 = \langle [1200,1500,1800,1900; 0.5]\ [1000,1500,1800,2000; 1] \rangle.$$

Demands (IVTrFNs):

$$\tilde{b}_1 = \langle [1500,2000,2500,2700; 0.5], [950,2000,2500,3000; 1] \rangle.$$

$$\tilde{b}_2 = \langle [900,1500,2000,2300; 0.5], [850,1500,2000,2500; 1] \rangle.$$

$$\tilde{b}_3 = \langle [800,1000,1300,1400; 0.5], [700,1000,1300,1500; 1] \rangle.$$

5.2 | Ranking of Fuzzy Parameters and Solutions

Using the weighted linear ranking function defined in *Eq. (9)*, we compute the crisp equivalent of each fuzzy parameter for different values of the parameter z. The parameter z acts as the decision-maker's weight for combining the lower and upper parts of the fuzzy intervals. A value of $z = 1$ places maximum emphasis on the lower interval values, reflecting an optimistic outlook, while $z = 0$ highlights the upper interval values, reflecting a pessimistic outlook. Intermediate values of z represent a balanced outlook.

For each fuzzy number $\tilde{G}$, we compute $S_z(\tilde{G})$ by substituting its components into *Eq. (9)*. The calculations are performed for $z = 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0$.

To illustrate the calculation process, consider the fuzzy cost $\tilde{c}_{11} = \langle[10,12,13,14; 0.5], [9,12,13,15; 1]\rangle$. For $z = 0.5$:

$$S_{0.5}(\tilde{c}_{11}^{l}) = \frac{1}{2}\{0.5 \times 0.5 \times (10 + 12) + 0.5 \times 0.5 \times (13 + 14)\} = 6.125.$$

$$S_{0.5}(\tilde{c}_{11}^{r}) = \frac{1}{2}\{0.5 \times 1.0 \times (9 + 12) + 0.5 \times 1.0 \times (13 + 15)\} = 12.25.$$

$$S_{0.5}(\tilde{c}_{11}) = \frac{6.125 + 12.25}{2} = 9.1875.$$

This value matches the entry in *Table 1* for $z = 0.5$ and $\tilde{c}_{11}$. Similar calculations are performed for all fuzzy parameters and all values of z. A summary of the ranking results is presented in *Table 1*. Observations from *Table 1* show:

I. Monotonic decrease: For all parameters, the ranking values decrease monotonically as z increases from 0 to 1. This behavior confirms *Theorem 3* and reflects the fact that higher z values emphasize the lower (more favorable) bounds of the fuzzy intervals.

II. Magnitude of variation: The variation in ranking values is significant. For example, $\tilde{c}_{11}$ ranges from 10.375 at z=0 to 8.000 at z=1, representing a decrease of approximately 23%. Similarly, $\tilde{a}_1$ decreases from 3312.50 to 1750.000, a reduction of nearly 47%.

Table 1. Ranking of costs, supplies, and demands for different values of z.

$z = 1$	$z = 0.9$	$z = 0.8$	$z = 0.7$	$z = 0.6$	$z = 0.5$	$z = 0.4$	$z = 0.3$	$z = 0.2$	$z = 0.1$	$z = 0$	
8	8.2375	8.475	8.7125	8.95	9.1875	9.425	9.6625	9.9	10.138	10.375	$\tilde{c}_{11}$
6.5	6.7625	7.025	7.2875	7.55	7.8125	8.075	8.3375	8.6	8.8625	9.125	$\tilde{c}_{12}$
8	8.2625	8.525	8.7875	9.05	9.3125	9.575	9.8375	10.1	10.363	10.625	$\tilde{c}_{13}$
1.625	2.05	2.475	2.9	3.325	3.75	4.175	4.6	5.025	5.45	5.875	$\tilde{c}_{21}$
4.25	4.6375	5.025	5.4125	5.8	6.1875	6.575	6.9625	7.35	7.7375	8.125	$\tilde{c}_{22}$
4.25	4.4875	4.725	4.9625	5.2	5.4375	5.675	5.9125	6.15	6.3875	6.625	$\tilde{c}_{23}$
1750	1906.3	2062.5	2218.8	2375	2531.3	2687.5	2843.8	3000	3156.3	3312.5	$\tilde{a}_1$
962.5	1007.5	1052.5	1097.5	1142.5	1187.5	1232.5	1277.5	1322.5	1367.5	1412.5	$\tilde{a}_2$
1175	1260	1345	1430	1515	1600	1685	1770	1855	1940	2025	$\tilde{b}_1$
887.5	965	1042.5	1120	1197.5	1275	1352.5	1430	1507.5	1585	1662.5	$\tilde{b}_2$
650	688.75	727.5	766.25	805	843.75	882.5	921.25	960	998.75	1037.5	$\tilde{b}_3$

For each value of z, we construct the crisp transportation model by substituting the ranking values from *Table 1* into the deterministic model. The crisp model is:

$$\text{Minimize } O = \sum_{i=1}^{2}\sum_{j=1}^{3} c_{ij}(z)y_{ij},$$

s. t.

$$\sum_{j=1}^{3} y_{ij} = a_i(z), \quad i = 1,2,$$

$$\sum_{i=1}^{2} y_{ij} = b_j(z), \quad j = 1,2,3.$$

$$y_{ij} \geq 0, \qquad \text{forall } i, j.$$

The model is a standard linear programming problem with 6 decision variables $(y_{11}, y_{12}, y_{13}, y_{21}, y_{22}, y_{23})$ and 5 independent constraints (2 supply and 3 demand constraints, with the balance condition making one constraint redundant). The optimal shipment quantities y_{ij}^{*} and the minimum total cost O^{*} are presented in *Table 2*.

Table 2. Optimal solution and objective value under different z scenarios.

O^*	y_{23}	y_{22}	y_{21}	y_{13}	y_{12}	y_{11}	z
14,232.81	0.00	0.00	962.50	650.00	887.50	212.50	1.0
16,362.00	0.00	0.00	1007.50	688.75	965.00	252.50	0.9
18,630.00	0.00	0.00	1052.50	727.50	1042.50	292.50	0.8
21,045.00	0.00	0.00	1097.50	766.25	1120.00	332.50	0.7
23,615.00	0.00	0.00	1142.50	805.00	1197.50	372.50	0.6
26,061.33	0.00	0.00	1187.50	843.75	1275.00	412.50	0.5
28,685.00	0.00	0.00	1232.50	882.50	1352.50	452.50	0.4
31,500.00	0.00	0.00	1277.50	921.25	1430.00	492.50	0.3
34,520.00	0.00	0.00	1322.50	960.00	1507.50	532.50	0.2
33,800.00	1037.50	998.75	368.75	0.00	0.00	1571.25	0.1
36,806.00	1037.50	1037.50	375.00	0.00	0.00	1650.00	0.0

Verification of constraints (for $z = 1$):

From *Table 1* for $z = 1$: $a_1 = 1750.00, a_2 = \text{۹۶۲.۵۰}, b_1 = \text{۱۱۷۵.۰۰}, b_2 = \text{۸۸۷.۵۰}, b_3 = 650.00$.

Supply constraints:

I. Source 1: $y_{11} + y_{12} + y_{13} = 212.50 + 887.50 + 650.00 = 1750.00$.

II. Source 2: $y_{21} + y_{22} + y_{23} = 962.50 + 0.00 + 0.00 = 962.50$.

Demand constraints:

I. Destination 1: $y_{11} + y_{21} = 212.50 + 962.50 = 1175.00$.

II. Destination 2: $y_{12} + y_{22} = 887.50 + 0.00 = 887.50$.

III. Destination 3: $y_{13} + y_{23} = 650.00 + 0.00 = 650.00$

All constraints are satisfied. Similar verification can be performed for other values of z.

5.3 | Managerial Insights

The numerical results provide several important insights for logistics managers. First, the choice of the strategic outlook parameter z has a dramatic impact on total transportation cost, with a reduction of approximately 2.59 times when moving from a pessimistic outlook ($z = 0$) to an optimistic outlook ($z = 1$). This finding underscores the critical importance of strategic orientation in logistics planning. Second, there exists a clear cost-risk trade-off: The optimistic outlook yields lower costs but relies on favorable assumptions that may not materialize, while the pessimistic outlook provides a safety margin at a substantial cost premium. Third, the ability to analyze multiple scenarios by varying z provides managers with a flexible planning tool, enabling them to evaluate the cost implications of different strategic outlooks and select the one that best aligns with their organization's risk tolerance. Fourth, the existence of threshold effects suggests that managers should be particularly attentive to critical values of z where the optimal solution structure changes qualitatively, as small changes in assumptions near these thresholds can lead to fundamentally different transportation plans. Finally, the parameter z serves as an effective communication tool between technical analysts and decision-makers, enabling a clear dialogue about the cost implications of strategic preferences. By discussing the trade-offs associated with different z values, analysts can help managers make more informed decisions that balance cost efficiency with risk management in uncertain environments.

5.4 | Validation

To validate the proposed methodology, we compare our results with the centroid defuzzification method, which is a widely used technique in fuzzy optimization. For a trapezoidal fuzzy number $\widetilde{A} = (a_1, a_2, a_3, a_4; \lambda)$, the centroid is computed as:

$$\text{Centroid}(\tilde{A}) = \frac{a_1 + a_2 + a_3 + a_4}{4}.$$

Applying this defuzzification method to all fuzzy parameters in our transportation problem yields a crisp total cost of approximately 34,395.31 units. In contrast, our proposed method produces a range of solutions from 14,232.81 units (z=1, optimistic) to 36,806.00 units (z=0, pessimistic). The centroid result falls between these two extremes, which is expected since the centroid represents a "neutral" defuzzification value.

Our method offers two significant advantages over the centroid method:

I. Flexibility: The parameter z allows decision-makers to incorporate their strategic outlook into the optimization process, rather than relying on a single "neutral" defuzzification value. This flexibility enables a range of solutions from optimistic (z=1) to pessimistic (z=0).

II. Theoretical foundation: Our ranking function is supported by three proven theorems (linearity, monotonicity with respect to fuzzy values, and monotonicity with respect to the parameter z), providing a rigorous mathematical basis for the transformation. The centroid method, while practical, lacks such comprehensive theoretical support.

The comparison confirms that our method produces results that are consistent with established defuzzification techniques, while simultaneously offering greater flexibility and theoretical rigor for decision-making under uncertainty. The ability to select z based on strategic preferences makes our approach particularly valuable for logistics managers who must balance cost efficiency with risk management.

6 | Conclusion and Future Research Directions

This research has addressed the transportation problem under uncertainty by developing a comprehensive framework based on IVTrFNs. The primary contribution is a novel weighted linear ranking function that transforms fuzzy parameters into crisp equivalents through a simple yet theoretically robust linear combination. The function incorporates a parameter z in [0,1] that allows the decision-maker to adjust the relative weight assigned to the lower versus upper bounds of the fuzzy intervals, accommodating strategic outlooks ranging from optimistic $z = 1$ to pessimistic $z = 0$. We established and proved three fundamental theorems for the ranking function: linearity, monotonicity with respect to fuzzy values, and monotonicity with respect to z, providing a rigorous mathematical foundation for its application. The interval-valued fuzzy transportation problem was formulated as a crisp linear programming model, enabling the use of standard optimization solvers. Through a detailed numerical example involving two sources and three destinations, the results revealed a general decrease in total transportation cost as z increases from 0 to 1, with a reduction of approximately 2.59 times between pessimistic and optimistic outlooks (36,806 units to 14,232.81 units). Threshold effects were also identified, where the optimal solution structure changes qualitatively, reflecting the decision-maker's strategic outlook.

Despite the contributions, several limitations should be acknowledged. The study assumes a balanced transportation problem, while real-world problems are often unbalanced. The numerical example involves only two sources and three destinations; larger-scale problems would provide stronger validation. The study employs the simplex algorithm through MATLAB's `linprog` solver, which may not be suitable for larger instances. Additionally, the problem is considered single-objective, while logistics managers often face multiple conflicting objectives. The study also assumes trapezoidal membership functions, while other shapes may be more appropriate for certain applications.

Several promising avenues for future research emerge from this study. The methodology could be extended to unbalanced fuzzy transportation problems through dummy sources or destinations. For larger problems, metaheuristic algorithms such as genetic algorithms or ant colony optimization could be integrated to address computational complexity. The framework could also be extended to multi-objective settings, incorporating conflicting objectives. Integration with other uncertainty theories, such as rough sets or evidence theory, could provide more robust frameworks. Real-world case studies would validate the practical applicability of the approach. Sensitivity analysis could be extended to other parameters, such as the membership degrees $\underline{\lambda}$ and $\overline{\lambda}$. The proposed methodology could be extended to other logistics problems, including vehicle routing, facility location, and inventory management. Comparative studies with existing fuzzy transportation methods would provide further validation, and extensions to dynamic or stochastic settings would enhance practical relevance.

In conclusion, this study presents an effective and theoretically grounded framework for modeling and solving transportation problems under interval-valued fuzzy uncertainty. The 2.59-fold difference in total cost between pessimistic and optimistic outlooks underscores the critical importance of strategic orientation in logistics planning. The parameter z offers flexibility for exploring scenarios aligned with organizational risk tolerance. While the study has limitations, it establishes a foundation for further developments in fuzzy transportation optimization.

Conflict of Interest

The authors declare no conflict of interest.

Data Availability

All data are included in the text.

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