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An Installment Purchase System and Its Comparison in a Deteriorating Economic Order Quantity Model

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Abstract

In this paper, we have proposed an EOQ model under two-level trade credit with installments specially the Equal Monthly Investment (EMI) policies. Installment is a financing arrangement that increases the sales for the business manager and it is very much beneficial for the consumer, specially for the higher value product. Here, the supplier offers a partial credit period M and the retailer offers a permissible delay period N to its customers. The customer pays a fraction α of the amount at the time of purchasing the product and rest $(1 - \alpha)$ by R_1 installments within the time N . This study investigates the cases and subcases in details under the conditions $N \leq M$ and $N \geq M$ respectively and generalizes some of the literatures such as Soni and Joshi (2013), Mahata and Mahata (2011), Teng (2009) etc when the number of installment is one. We have developed a profit maximization problem on retailer's inventory model for different cases and subcases. Also, we have shown that the average profit maximizes with installment purchase system than delay-in-payment system and increases with more down payment. Finally, numerical illustrations and its comparative study with the existing methods are performed to justify the novelty of the proposed approach.

Keywords: Inventory, Economic order quantity, Partial trade credit, Installment.

1 | Introduction

In recent times, firms and managers are adopting new techniques and strategies to boost their sales and to attract more business. One of such strategy is “Buy Now and Pay Later”(BNPL) in which the supplier usually offers a credit period to the retailer and the retailer can collect the money in a flexible way. Atradius [1] showed that in some european countries companies give 80% trade credits of their business transactions. Klapper et al. [2] analyzed that 90% of world merchandise was financed based on trade credits in 2007. Nowadays, trade

credit plays an important role and becomes a common practice for the organizers. There exists two kinds of trade credit policies, 1) partial trade credit policy, and 2) full trade credit policy. In partial trade credit policy the supplier offers a credit period to the retailer where the retailer pays a fraction of amount when the order is placed and the rest of he is paying within the credit period whereas in full trade credit, the supplier gives a full credit period on the entire purchased amount. Also, if the supplier is only offering the credit period to its retailer, it is called one level credit period and if the retailer is also giving a credit period to its customers, it is called two level credit period.

Since Goyal [3] study of trade credit on a basic inventory model, many researchers have worked in this direction. Teng [4] analyzed an EOQ model with full trade credit policy. Jaggi and Verma [5] presented partial trade credit in an EOQ model which generalizes many models such as Goyal [3], Teng [4] etc. Mahata and Mahata [6] considered the partial trade credit offered by the retailer in a deteriorating economic production quantity model. Soni and Joshi [7] extended Mahata's work and proposed a more general model two-level partial trade credit policy in a deteriorating EOQ model. Yu [8] considered delay in payment in a supplier retailer integrated inventory model. Ouyang et al. [9] examined the trade credit in a deteriorating model. Yang et al. [10] examined a production-inventory under two-part trade credit. Later on, Ting [11] revisited that model.

De et al. [12] developed an inventory model where they considered the carbon emission from manufacturing system and the sustainable trade credit financing for deteriorating items. Khan et al. [13] proposed an EOQ model under hybrid prepayment and delay payment. Also, Duany et al. [14] considered the advance payment and delay in payment together in an inventory model. Amiri et al. [15] investigated inventory model with advance payment via evidence theory. Shah and Shroff [16] proposed an inventory model with two-level trade credit for trapezoidal demand.

However, trade credit has also been analyzed in supply chain models. Earlier researchers have shown that the order quantity varies with trade credit. It is often seen that customers order more if trade credit is given [17]. Heydari [18] studied trade credit also helps align the interests of the parties. Later, researchers investigated a supply chain model with the credit dependent demand [19]. Tiwari et al. [20] considered two-level partial trade credit in supply chain models. Sarkar et al. [21] considered the joint effect of carbon emission and delay in payment in a supply chain model for multiple retailers. Market demand is dependent on seller's credit.

On that note, Zhang et al. [22] develop a supply chain model using Stackelberg approach. Trade credit also affects demand. Marchiet al. [23] developed two-level delay in payment in a vendor-buyer model where the demand is dependent on credit period. Seifbarghy et al. [24] studied a two-echelon supply chain where they have shown that the supply chain can use the benefits of economies of scale. Moeany et al. [25] discussed the refunding and bundling reservation in a two-layer supply chain. Cao et al. [26] examined the best payment scheme for the supply chain members. Recently, Gao [27] studied the impact of green credit policy on trade credit system. Lai et al. [28] investigated the firm's performance after trade credit after natural disaster. Priya et al. [29] explored the impact of return policies on trade credit system.

Moreover, in practice inventory items like fruits, vegetables, medicines and another chemical etc. began to deteriorate over time. Ghare and Schrader [30] first introduced the concept of deterioration in a classical EOQ model with exponentially decaying inventory for a constant demand. Thereafter several research papers have been published in this direction [7], [10], [31–33]. In order to reduce the cost due to deterioration, any kinds of firms may need to backlog demand.

So, deterioration and backlogging are complementary conditions for real practices. A large number of inventory models available in the literature in which the authors' focus is the backlogging rate depending on a number of customers in the waiting time. Montgomery et al. [34], Abad [35], Karmakar et al. [36], Karmakar et al. [37], Choudhuri et al. [38], De et al. [12] developed inventory models with backorders and lost sales. Ghosh et al. [39] assumed partial backlog-ging that depends on the waiting time. Mondal et al. [40] investigated a supplier-retailer-customer supply chain model where the supplier adopts a full credit policy for the retailer.

In this article, we formulate a deteriorating EOQ model with installments in a partial trade credit system. Here the supplier gives a credit period of M time to the retailer, the customer pays a fraction α of the purchasing cost as a down payment and the rest $(1 - \alpha)$, is paid by R_1 number of equal installment within the time period N . We have assumed the demand function dependent on selling price. Considering these circumstances, we model the retailer's inventory system as a profit maximization problem. Numerical examples are presented to illustrate the proposed model and the effect of key parameters on the optimal solution is studied.

2 | Background of the Study

Under an installment credit system, the retailer allows the consumer to pay off the purchasing cost in small increments over time instead of all at once (*Fig. 1*). The customer can repay the loan in several months or years, depending on the repayment terms or the number of installments. It enables the customer to avoid repaying a large amount of debt at once, which can be difficult to manage especially for those who run a household and have tight budgets to follow. By taking advantage of different BNPL payment schemes, people have more leeway to spend without the fear of getting into any major financial setback. According to a survey by "Australian Securities and Investment Commission" taken in March 03, 2021, the total number of BNPL users are 3748737 in the year 2018-2019.

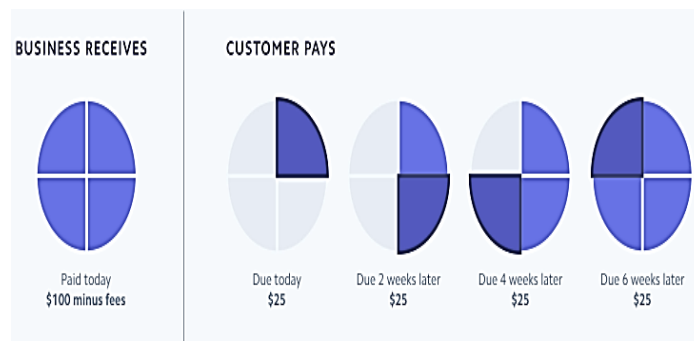


Fig. 1. Installments purchase system.

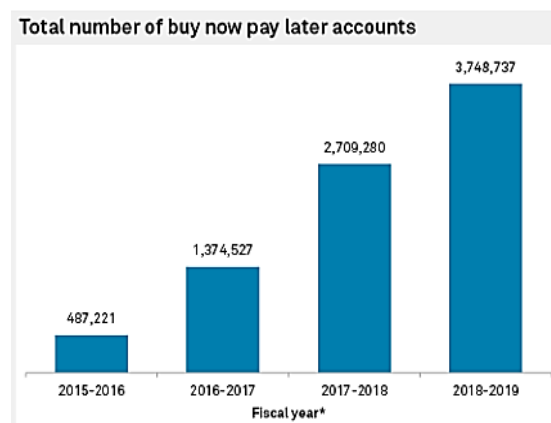


Fig. 2. Total number of BNPL accounts.

Installment purchase system offers some benefits (*Fig. 3*) to both the installment seller and installment purchaser. An installment loan offers for the business is the potential to increase sales. Customers can purchase items or services they might not otherwise afford in a single lump sum. Also, installment purchase system provides flexible terms and conditions which help to attract more customers to the business. In addition, customer satisfaction is one of the prime aim of business managers. Installment loans can help to strengthen customer relationships by giving current customers access to funds. It also helps the business to stay more competitive in the marketplace.



Fig. 3. Total number of BNPL accounts.

In installment purchase system, the purchaser acquires an asset from the installment seller by signing an agreement and agrees to pay the price-a fraction as a down payment and the rest of the amount by certain number of installments with interest as agreed to the seller. The following illustrates how the installment purchase system works in a real life setting:

- I. Under installment purchase agreement, the ownership of the goods passes immediately to the buyer after signing the agreement by the parties.
- II. Since the ownership passes to the buyer of goods, (s)he is responsible for any loss to the asset and such loss should be borne by the buyer only.
- III. According to this system, the buyer has a right to dispose off such
- IV. asset, since (s)he is the owner of it.
- V. If the purchaser makes any default on an installment payment, the seller has no right to repossess the asset but (s)he can only sue the buyer for recovery of the outstanding amount with interest also.
- VI. The buyer of asset has no right to return such an asset or to terminate the agreement of installment purchase but (s)he is liable for payment of the remaining installments as mentioned in the agreement.

3 | Problem Definition

The main goal of this work is to investigate the installment purchase system by extending the work of Soni and Joshi [7] and analyzing the payment scheme. In the installment purchase system, the customer needs to make a down payment, a percentage of the selling price. Then the seller/installment provider offers a credit period. During this period, the rest of the selling price will be paid by a number of installments as per customer's choice. Based on this situation, we define the basic problem as:

What will be the optimum order quantity(Q) in total cycle time (T) so that the business manager can maximize the total average profit with respect to the credit periods offered by the supplier (M) and the retailer (N) and the number of installments (R_1)?

4 | Model Development

4.1 | Notations and Assumptions

To develop the proposed model, the following notations are used:

- I. P_r Purchasing cost of the retailer.
- II. α Customer's fraction of the total amount owed payable at the time of.
- III. placing an order within the delay period to the retailer, where $0 \leq \alpha \leq 1$.

- IV. Q Ordering quantity.
- V. D Demand rate is assumed to be known and constant.
- VI. θ Constant deterioration rate per unit time per unit item.
- VII. T Length of replenishment cycle.
- VIII. h_v Supplier's holding cost per item per year.
- IX. h_{b1} Retailer's holding cost for defective items per item per year.
- X. h_{b2} Supplier's holding cost for non-defective items per item per year.
- XI. T Cycle time.
- XII. M The retailer's trade credit period offered by the supplier in years.
- XIII. N The retailer's trade credit period offered by the retailer in years.
- XIV. I_e Interest earned.
- XV. I_c Interest charged.
- XVI. P_c Purchasing price of customer.
- XVII. R_1 Number of installments given by the retailer to the customers.

We develop the model with the following assumptions:

- I. Replenishments are instantaneous and the lead time is zero.
- II. Shortages are not allowed.
- III. Time horizon is infinite.
- IV. During the credit period offered by the supplier, the retailer uses the sales revenue to earn interest at a rate I_e . At the end of the permissible delay period, the retailer pays the purchasing cost to the supplier and pays interest charges at a rate I_c for the items in stock or the items already sold but have not paid yet.

5 | Formulation of the Model

5.1 | Sales Purchase System

Let $q(t)$ be the inventory level at any time t ($0 \leq t \leq T$). Initially, the stock level is Q . The inventory level decreases due to both demand and deterioration both until it becomes zero at time $t = T$. The differential equation governing the system in the interval $(0, T)$ is

$$\frac{Dq(t)}{dt} + \theta q(t) = -D, 0 \leq t \leq T. \quad (1)$$

$$q(t) = \frac{D}{\theta} (e^{\theta(T-t)} - 1), 0 \leq t \leq T. \quad (2)$$

Using the boundary condition $q(0) = Q$, the order quantity can be obtained as

$$Q = \frac{D}{\theta} (e^{\theta T} - 1). \quad (3)$$

The total demand during one cycle is DT . The number of units deteriorated during one cycle can be expressed as

$$Q - DT = \frac{D}{\theta} (e^{\theta T} - 1 - \theta T). \quad (4)$$

Thus, the deterioration cost, $DC = \frac{cD}{\theta T}(e^{\theta T} - 1 - \theta T)$ Holding cost, $HC = \frac{hD}{\theta^2 T}(e^{\theta T} - 1 - \theta T)$ Ordering cost, $C = \frac{A}{T}$ Sales Profit, $SP = (s - c)D$. Therefore, the total average profit, $\Pi_s = SP - OC - HC - DC$, i.e.,

$$\Pi_s = (s - c)D - \frac{A}{T} - \frac{D}{\theta^2 T}(h + \theta c)(e^{\theta T} - 1 - \theta T). \quad (5)$$

In this purchase system, instead of giving the whole payment at the time of purchasing the product, the supplier offers a partial credit period M to the retailer whereas the retailer asks his customer to pay a fraction α of the purchasing cost when a order is placed as a down payment and the rest fraction of amount, $(1 - \alpha)$, of the purchasing cost by R_1 installments within a credit period N . Based on the values of M and N , two potential cases are considered: $N \leq M$ and $M \leq N$.

5.2.1 | Case 1: $N \leq M$

In this case, the length of credit period M offered by the supplier is larger than the length of credit period N offered by the retailer. For the retailer, the total profit per unit time is composed of sales profit, ordering cost, holding cost, deterioration cost, interest earned and interest paid.

Based on the values of M (i.e. the time at which the retailer must pay the supplier to avoid interest charge), T (i.e. the replenishment cycle time), $T + N$ (i.e. the time at which the retailer receives the payment from the last customer), we have to examine following three situations: Cycle, IE2, is

$$0 < (T + N) \leq M. \quad (5.1)$$

$$T \leq M \leq (T + N). \quad (5.2)$$

$$M \leq T. \quad (5.3)$$

5.2.1.1 | $0 < (T + N) \leq M$

In this case, the retailer receives all returns from the customers before paying the purchase amount to the supplier. Consequently, the retailer does not incur interest charges in this case. So, the interest earned per cycle, IE1, is calculated as; 1) interest from the down payments in $[0, T]$ and $[M, T]$, 2) interests from each installments in $[\frac{N}{R_1}, N]$, 3) interest from $(1 - \alpha)$ amount in $[N, T + N]$ and $[T + N, M]$.

$$IE1 = \frac{I_e}{T} \left[\left\{ \int_0^T \alpha s D t dt + \alpha s D T (M - T) \right\} + \int_N^{T+N} \frac{R_1 (1 - \alpha) s D}{R_1} (t - N) dt + (1 - \alpha) s D T (M - T - N) \right] + IE11. \quad (6)$$

where the interest from installments is given by

$$IE11 = \int_{\frac{N}{R_1}}^{\frac{2N}{R_1}} \frac{(1 - \alpha) s D}{R_1} \left(t - \frac{N}{R_1} \right) dt + \int_{\frac{2N}{R_1}}^{\frac{3N}{R_1}} \frac{2(1 - \alpha) s D}{R_1} \left(t - \frac{2N}{R_1} \right) dt + \dots + \int_{\frac{(R_1 - 1)N}{R_1}}^N \frac{(R_1 - 1)(1 - \alpha) s D}{R_1} \left(t - \frac{(R_1 - 1)N}{R_1} \right) dt. \quad (7)$$

5.2.1.2 | $T \leq M \leq (T + N)$

During this period M , the retailer's revenue comes from; 1) interest from the down payment, 2) interest from installments (IE11), and 3) interest from the portion of delayed payments in $[N, M]$. Therefore, the interest earned per cycle, IE2, is

$$= \frac{I_e}{T} \left[\int_0^T \alpha s D t dt + \alpha s D T (M - T) + IE11 + \int_N^M (1 - \alpha) s D (t - N) dt \right], \quad (8)$$

where IE11 is given by Eq. (7) In this subcase, the retailer has to pay for the items sold after time M , so the interest charged per cycle, IC2

$$= \frac{cI_c}{sT} \left[\int_M^{T+\frac{iN}{R_1}} \frac{(1 - \alpha)sD}{R_1} (t - M) dt + \int_{T+\frac{iN}{R_1}}^{T+\frac{(i+1)N}{R_1}} \frac{2(1 - \alpha)sD}{R_1} \left(t - \left\{ T + \frac{iN}{R_1} \right\} \right) dt + \dots \right. \\ \left. + \int_{T+\frac{(R_1-1)N}{R_1}}^{T+N} \frac{(R_1 - i)(1 - \alpha)sD}{R_1} \left(t - \left\{ T + \frac{(R_1 - 1)N}{R_1} \right\} \right) dt \right]. \quad (9)$$

5.2.1.3 | $M \leq T$

In this time interval, the retailer's revenue comes from; 1) interest from the down payment, 2) interest from installments (IE11), 3) interest from the portion of delayed payments in $[N, M]$. So, the interest earned per cycle, IE3

$$= \frac{I_e}{T} \left[\int_0^M \alpha s D t dt + IE11 + \int_N^M (1 - \alpha) s D (t - N) dt \right]. \quad (10)$$

Here, the retailer has to pay the down payment of the items sold after time M and credit payment for items sold after $M - N$. Hence, the interest charged per cycle, IC3

$$= \frac{cI_c}{sT} \left[\int_M^T \frac{\alpha s D}{\theta} (e^{\theta(T-t)} - 1) dt \right] + IC11. \quad (11)$$

Thus, the total average profit for case I ($M \leq T$) is given by

$$\pi_I^{(1)} = \begin{cases} \pi_{1I} & \text{if } 0 < T \leq M - N, \\ \pi_{2I} & \text{if } M - N \leq T \leq M, \\ \pi_{3I} & \text{if } M \leq T, \end{cases} \quad (12)$$

where $\pi_{iI} = SP - OC - HC - DC + IE_i - IC_i$, $i = 1, 2, 3, 4, 5$

$$\pi_{1I} = (s - c)D - \frac{A}{T} - \frac{D}{\theta^2 T} (h + \theta c) (e^{\theta T} - 1 - \theta T) + \frac{sI_c D}{2} (T + 2\alpha(M - T) + 2(1 - \alpha)(M - T - N)) + \frac{I_e}{T} IE11. \quad (13)$$

Similarly,

$$\pi_{2I} = (s - c)D - \frac{A}{T} - \frac{D}{\theta^2 T} (h + \theta c) (e^{\theta T} - 1 - \theta T) \\ + \frac{sI_c D}{2T} (\alpha T^2 + 2\alpha T(M - T) + (1 - \alpha)(M - N)^2) - \frac{(1 - \alpha)cI_c D}{2T} (T + N - M)^2 + \frac{I_c}{T} IE1 - IC1. \quad (14)$$

And

$$\begin{aligned}
\pi_{3I} = & (s - c)D - \frac{A}{T} - \frac{D}{\theta^2 T} (h + \theta c)(e^{\theta T} - 1 - \theta T) + \frac{sI_c D}{2T} (\alpha M^2 + (1 - \alpha)(M - N)^2) \\
& - \frac{cI_c D}{T} \left(\frac{\alpha}{\theta^2} (e^{\theta(T-M)} - \theta(T-M) - 1) + \frac{(1 - \alpha)}{2} (T + N - M)^2 \right) \\
& + \frac{I_c}{T} IE1 - IC1.
\end{aligned} \tag{15}$$

5.2.2 | Case 2: $M \leq N$

Based on the values of M and T , we explore the following two situations:

$$(5.2.2.1) T \leq M,$$

$$(5.2.2.2) M \leq T.$$

5.2.2.1 | $T \leq M$

The retailer is gaining interests, 1) from the down payments of the items sold in $[0, T]$, 2) interests from installments, and 3) interests from the down payments in $[T, M]$. Therefore, the interest earned per cycle, $IE4$

$$= \frac{I_e}{T} \left[\int_0^T \alpha s D t dt + IE41 + \alpha s D T (M - T) \right]. \tag{16}$$

$$\begin{aligned}
IE41 = & \int_{\frac{N}{R_1}}^{\frac{2N}{R_1}} \frac{(1 - \alpha)sD}{R_1} \left(t - \frac{N}{R_1} \right) + \int_{\frac{2N}{R_1}}^{\frac{3N}{R_1}} \frac{2(1 - \alpha)sD}{R_1} \left(t - \frac{2N}{R_1} \right) dt \\
& + \int_{\frac{(i-1)N}{R_1}}^M \frac{(i-1)(1 - \alpha)sD}{R_1} \left(t - \frac{(i-1)N}{R_1} \right) (M - i - 1).
\end{aligned} \tag{17}$$

Interest charged per cycle, $IC4 = \frac{dI_c}{dT} (IC22 + IC23)$, where $IC22$, portion of $(1 - \alpha)$ not paid by installments and $IC23$, items sold but some portion from the remaining installments, are given below

$$\begin{aligned}
IC22 = & \int_N^{T + \frac{i_1 N}{R_1}} \frac{(1 - \alpha)sD}{R_1} \left(T + \frac{i_1 N}{R_1} - t \right) dt + \dots \\
& + \int_{T + \frac{(n_1 - 1)N}{R_1}}^{T + N} \frac{(R_1 - i_1)(1 - \alpha)sD}{R_1} (T + N - t) dt.
\end{aligned} \tag{18}$$

$$\begin{aligned}
IC23 = & \int_M^{\frac{N}{R_1}} \left(\frac{iN}{R_1} - M \right) (1 - \alpha)sD + \int_{\frac{(i+1)N}{R_1}}^{\frac{(i+2)N}{R_1}} \left(\frac{iN}{R_1} - M \right) (1 - \alpha)sD + \dots \\
& + \int_{\frac{(R_1 - 1)N}{R_1}}^N \left(\frac{iN}{R_1} - M \right) (1 - \alpha)sD.
\end{aligned} \tag{19}$$

5.2.2.2 | $M \leq T$

Here, the interest earned per cycle, $IE5$, calculated as; 1) interests from down payments sold in $[0, M]$ and, 2) interest s from installments, is $\frac{L_4}{T} \left[\int_0^M \alpha s D t dt + IE25 \right]$ where,

$$\begin{aligned}
 IE25 = & \left[\int_{\frac{N}{R_1}}^{\frac{2N}{R_1}} \frac{(1-\alpha)sD}{R_1} \left(t - \frac{N}{R_1} \right) + \int_{\frac{2N}{R_1}}^{\frac{3N}{R_1}} \frac{2(1-\alpha)sD}{R_1} \left(t - \frac{2N}{R_1} \right) dt \right. \\
 & \left. + \dots + \int_{\frac{(i-1)N}{R_1}}^{\frac{iN}{R_1}} \frac{(i-1)(1-\alpha)sD}{R_1} \left(t - \frac{(i-1)N}{R_1} \right) dt \right].
 \end{aligned} \tag{20}$$

Interest charged per cycle, IC5, is 1) items sold after M, 2) some portion of $(1-\alpha)$ not paid by installments, 3) items sold but some portion from the remaining installments

$$IC5 = \frac{cI_c}{sT} \left[\int_M^T \frac{\alpha sD}{\theta} (e^{\theta(T-t)} - 1) dt + IC22 + IC23 \right]. \tag{21}$$

Thus, the total average profit for case 2 ($M \leq N$) is given by

$$\pi_I^{(2)} = \begin{cases} \pi_{4I} & \text{if } T \leq M, \\ \pi_{5I} & \text{if } T \geq M, \end{cases} \tag{22}$$

where

$$\begin{aligned}
 \pi_{4I} = & (s-c)D - \frac{A}{T} - \frac{D}{\theta^2 T} (h + \theta c) (e^{\theta T} - 1 - \theta T) + \frac{\alpha s I_e D}{2} (T + 2(M-T)) \\
 & - \frac{(1-\alpha)cI_c D}{2} (T + 2(N-M)) + \frac{I_e}{T} IE4 - IC4.
 \end{aligned} \tag{23}$$

$$\begin{aligned}
 \pi_{5I} = & (s-c)D - \frac{A}{T} - \frac{D}{\theta^2 T} (h + \theta c) (e^{\theta T} - 1 - \theta T) + \frac{\alpha s I_e D M^2}{2T} + \frac{I_e}{T} IE5 - IC5 \\
 & - \frac{cI_c D}{T} \left\{ \frac{\alpha (e^{\theta(T-M)} - \theta(T-M) - 1)}{\theta^2} + (1-\alpha) \left(T(N-M) + \frac{T^2}{2} \right) \right\}.
 \end{aligned} \tag{24}$$

Therefore, the total average profit is given by

$$\pi_I = \begin{cases} \pi_I^{(1)} & \text{if } N \leq M, \\ \pi_I^{(2)} & \text{if } N \geq M, \end{cases} \tag{25}$$

where $\pi_I^{(1)}$ and $\pi_I^{(2)}$ are given in Eqs. (12) and (22) respectively.

5.3 | Delay-in Payment

When the number of installment, $R_1 = 1$, $IE11 \rightarrow 0$. In the case of interest charges, $i = 1, i_1 = 1$ and $IC11 \rightarrow 0, IC22 \rightarrow 0, IC23 \rightarrow 0$. Then, the proposed model coincides with Soni and Joshi [7] model where the profit function is given by

$$\pi_D(T) = \begin{cases} \pi_D^{(1)}(T) & \text{if } N \leq M, \\ \pi_D^{(2)}(T) & \text{if } N \geq M, \end{cases} \tag{26}$$

Where

$$\pi_D^{(1)}(s, T) = \begin{cases} \pi_{1D}(s, T) & \text{if } 0 < T \leq M - N, \\ \pi_{2D}(s, T) & \text{if } M - N \leq T \leq M, \\ \pi_{3D}(s, T) & \text{if } M \leq T, \end{cases} \quad (27)$$

$$\pi_{1D}(s, T) = (s - c)D - \frac{A}{T} - \frac{D}{\theta^2 T} (h + \theta c)(e^{\theta T} - 1 - \theta T) + \frac{sI_e D}{2} (T + 2\alpha(M - T) + 2(1 - \alpha)(M - T - N)). \quad (28)$$

$$\pi_{2D}(s, T) = (s - c)D - \frac{A}{T} - \frac{D}{\theta^2 T} (h + \theta c)(e^{\theta T} - 1 - \theta T) + \frac{sI_e D}{2T} (\alpha T^2 + 2\alpha T(M - T) + (1 - \alpha)(M - N)^2) - \frac{(1 - \alpha)cI_c D}{2T} (T + N - M)^2. \quad (29)$$

$$\pi_{3D}(s, T) = (s - c)D - \frac{A}{T} - \frac{D}{\theta^2 T} (h + \theta c)(e^{\theta T} - 1 - \theta T) + \frac{sI_e D}{2T} (\alpha M^2 + (1 - \alpha)(M - N)^2) - \frac{cI_c D}{T} \left(\frac{\alpha}{\theta^2} (e^{\theta(T-M)} - \theta(T - M) - 1) + \frac{(1 - \alpha)}{2} (T + N - M)^2 \right). \quad (30)$$

And

$$\pi_D^{(2)}(s, T) = \begin{cases} \pi_{4D}(s, T) & \text{if } T \leq M, \\ \pi_{5D}(s, T) & \text{if } T \geq M. \end{cases} \quad (31)$$

$$\pi_{4D}(s, T) = (s - c)D - \frac{A}{T} - \frac{D}{\theta^2 T} (h + \theta c)(e^{\theta T} - 1 - \theta T) + \frac{\alpha sI_e D}{2} (T + 2(M - T)) - \frac{(1 - \alpha)cI_c D}{2} (T + 2(N - M)). \quad (32)$$

$$\pi_{5D}(s, T) = (s - c)D - \frac{A}{T} - \frac{D}{\theta^2 T} (h + \theta c)(e^{\theta T} - 1 - \theta T) + \frac{\alpha sI_e D M^2}{2T} - \frac{cI_c D}{T} \left\{ \frac{\alpha(e^{\theta(T-M)} - \theta(T - M) - 1)}{\theta^2} + (1 - \alpha) \left(T(N - M) + \frac{T^2}{2} \right) \right\}. \quad (33)$$

6 | Results

6.1 | Numerical Examples

In this section, a numerical analysis is presented to illustrate the behavior of the models developed for five different scenarios presented above. The input data used in this study are taken from Soni and Joshi [7]. The values for the parameters are: $A = 130, D = 3000, h = 7, c = 15, I_c = 0.15, I_e = 0.12, \theta = 0.1$. We compute the results for $\alpha = 0.4, M = 1, N = 0.4, R_1 = 12$ for case I ($N \leq M$) and $\alpha = 0.4, M = 0.8, N = 1, R_1 = 12$ for case II ($N \geq M$) with the help of LINGO-19 software and it is given in Table 1.

Table 1. Computational results of the total average profit for different purchase system.

Cases and Subcases	Decision Variables	Sales Purchase $\alpha = 1, R_1 = 1$	Installment Purchase $\alpha = 0.4, R_1 = 1222$	Delay-in-payment $\alpha = 0.4, R_1 = 1$
5.2.1.1	π_{11}	47457.82	45158.56	45038.62
	T	0.1599	0.1482	0.1599
	Q	483.72	447.89	483.72
5.2.1.2	π_{21}	45792.5	43404.10	43373.3
	T	0.6	0.6	0.6
	Q	1855.10	1855.10	1855.10

Table 1. Continued.

Cases and Subcases	Decision Variables	Sales Purchase $\alpha = 1, R_1 = 1$	Installment Purchase $\alpha = 0.4, R_1 = 1222$	Delay-in-payment $\alpha = 0.4, R_1 = 1$
5.2.1.3	π_{3x}	43778.49	41523.04	40783.79
	T	1	1	1.1657
	Q	3155.13	3155.13	3709.04
5.2.2.1	π_{4I}	45441.82	41212.88	41054.14
	T	0.1599	0.2454	0.1624
	Q	483.72	745.29	491.15
5.2.2.2	π_{5I}	42785.68	36553.39	38170.19
	T	0.8	0.8	0.8
	Q	2498.61	2498.61	2498.61

Here, we summarize the results of the installment purchase system model for different values of α and R_1 given in Table 2 and Table 3 respectively.

Table 2. Total average profit for different values of α for installment purchase system.

Cases and Subcases	Decision Variables	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 0.9$
5.2.1.1	π_{1I}	43631.32	45541.12	47073.99
	T	0.1398	0.1502	0.1580
	Q	422.34	454.05	477.93
5.2.1.2	π_{2I}	41811.83	43802.13	45394.43
	T	0.6	0.6	0.6
	Q	1855.10	1855.10	1855.10
5.2.1.3	π_{3I}	40019.63	41898.89	43402.29
	T	1	1	1
	Q	3155.13	3155.13	3155.13
5.2.2.1	π_{4I}	38763.95	41898.61	44722.87
	T	0.7721	0.2211	0.1680
	Q	2408.10	670.78	508.18
5.2.2.2	π_{5I}	35763.95	37592.10	41746.96
	T	0.8	0.8	0.8
	Q	2498.61	2498.61	2498.61

Table 3. Total average profit for different values of R_1 for installment purchase system.

Cases and Subcases	Decision Variables	$R_1 = 6$	$R_1 = 12$	$R_1 = 24$
5.2.1.1	π_{1I}	45264.29	45158.56	45100.17
	T	0.1378	0.1482	0.1539
	Q	416.31	447.89	465.33
5.2.1.2	π_{2I}	43429.3	43404.10	43389.4
	T	0.6	0.6	0.6
	Q	1855.10	1855.10	1855.10
5.2.1.3	π_{3I}	41524.07	41523.04	41521.46
	T	1	1	1
	Q	3155.13	3155.13	3155.13
5.2.2.1	π_{4I}	41245.68	41212.88	41199.29
	T	0.2376	0.2454	0.2486
	Q	721.41	745.29	755.18
5.2.2.2	π_{5I}	36556.37	36553.39	36564.51
	T	0.8	0.8	0.8
	Q	2498.61	2498.61	2498.61

7 | Discussions

Table 1 compares the total average profit for sales purchase system, installment purchase system and delay-in-payment for different cases and subcases. We have computed the results with 40% down payment and 12

number of installments for installment purchase system. For delay-in-payment, the customer has paid only 40% of the amount with 0.4 years credit period. We can see from subcase 5.2.1.1 that the profit for the installment (\$ 45158.56) is higher than the profit from delay-in-payment (\$45038.62). In subcase 5.2.1.1 the comparative study says that the profit for sales purchase is higher than any other system which is practical because the retailer is getting the whole amount instantly. Since the retailer is getting the payment from the last customer much earlier the time (s)he has to give to the supplier, so less cycle time (0.1482 years) and order quantity (447.89 units) are required for the installment purchase system than the other two systems.

From the subcase 5.2.1.2, the profit in installment purchase is greater than the profit in delay-in-payment though the cycle time and order quantity are same for three purchase system with 0.6 years and 1855.10 units respectively. In subcase 5.2.1.3 the order quantity in delay-in-payment (3709.04 units) is significantly high than the other two systems (3155.13 units) but the profit function behaves same as the previous subcases. Since the retailer has to pay the total amount to the supplier before the cycle time ends, (s)he requires a larger order quantity for delay-in-payment system because in installment purchase system at least (s)he is getting some amount. So, delay-in-payment system under this subcase is a bad choice for the retailer.

Under case 2, subcase 5.2.2.1, shows that the profit value (\$41212.88) in installment system is higher than the delay-in-payment (\$41054.14) but the cycle time (0.2454 years) and order quantity (745.29 units) are maximum than the other two systems because in installment purchase system the interest charged is higher than the delay-in-payment system. Only in subcase 5.2.2.2 we can see that the profit (\$38170.19) in delay-in-payment is higher than the profit (\$36553.39) in installment purchase system with equal order quantity (2498.61 units). This may occur because the interests earned from the installment is lesser than the interest charged in this time interval for the given set of values. Throughout the *Table 1*, it is observed that the total average profit is higher than the profit in delay-in-payments for all the subcases except subcase 5.2.2.2. The cycle time (1 year) is higher and the order quantity (3709.04 units) is maximum for the third subcase.

In *Table 2* we have given the results for $\alpha = 0$ (i.e. when the customer is not paying anything as a down payment), $\alpha = 0.5$ (i.e. when the customer pays 50% of the down payment) and $\alpha = 0.9$ (i.e. when the customer pays 90% as a down payment). From *Table 2* we examine that the total average profit is increasing with the increment of the down payment for all subcases. However, the cycle time and order quantity are either increasing or constant when the down payment is increasing. We can see a huge difference in the order quantity for 5.2.2.1 subcase.

For zero down payment ($\alpha = 0$) the order quantity is 2408.1 units for 0.7721 year cycle time whereas for 50% down payment the order quantity is 670.78 units for 0.2211 year cycle time. This means, to acquire \$38763.95 profit with zero payment, the retailer needs much more order quantity and longer cycle time but if (s)he has at least 50% payment, the struggle to get profit reduces, (s)he can start with less order quantity and lower cycle time.

Table 3 shows the nature of the profit function with the increment in the number of installments. For every subcases, the total average profit is decreasing as the number of installment is increasing except for $T \leq M$ under Case 2. Also, more order quantity is needed if number of installment is higher.

7 | Graphical Illustration

Here we draw several figures to justify the current approach of the proposed model. In the graphs, we have mentioned the subcases 5.2.1.1, 5.2.1.2, 5.2.1.3, 5.2.2.1, 5.2.2.2 as subcase I, II, III, IV, V respectively.

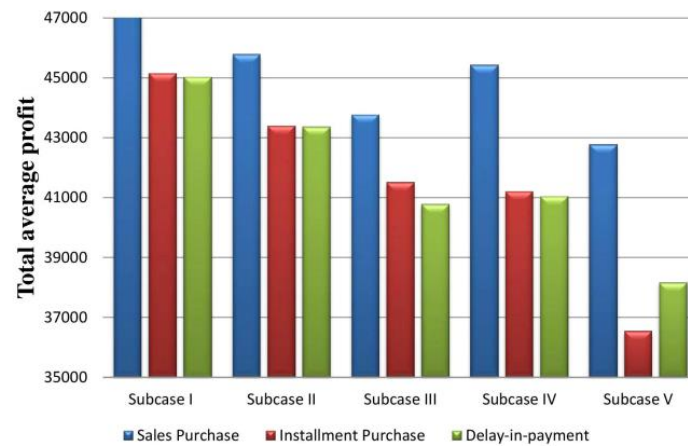


Fig. 4. Comparative study of the total average profit for different purchase system.

Fig. 4 compares the total average profit for different purchase system. In each of the scenarios the average profit for the sales purchase system is higher because the retailer is receiving the entire amount at the time of selling the product. But, if we compare the installment purchase and delay-in-payments system, the average profit is always higher except when the cycle time is greater than all the credit periods for both the supplier and retailer.

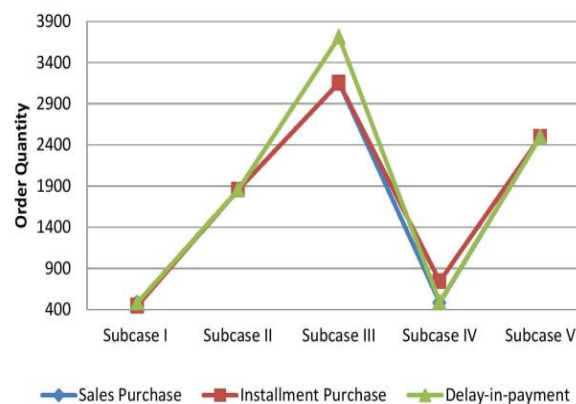


Fig. 5. Comparative study of the order quantity for different purchase system.

From Fig. 5 we have observed the variation in the order quantity for installment purchase system and delay-in-payment under subcase III and IV. Also, maximum order quantity is required for partial trade credit system without installment.

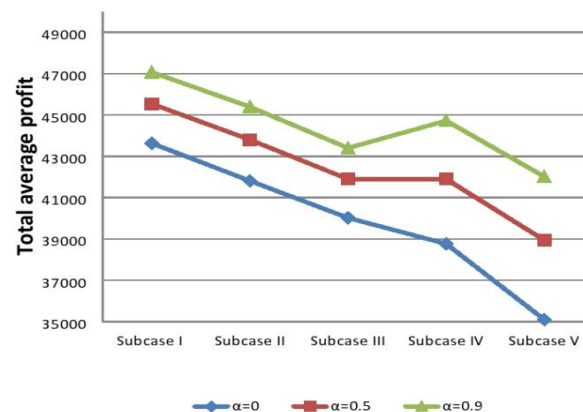


Fig. 6. Total average profit for different values of α

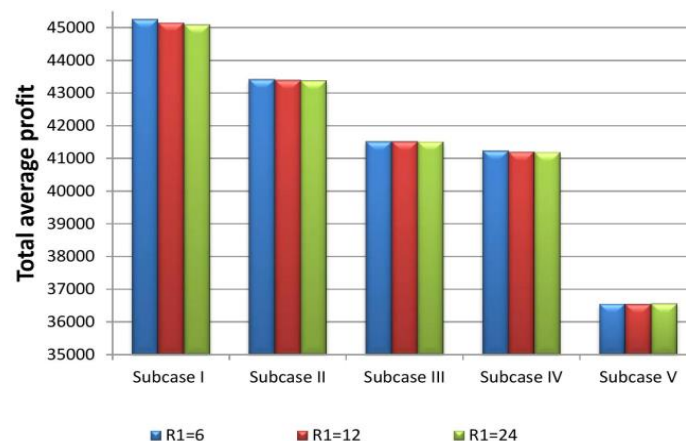


Fig. 7. Total average profit for different values of R_1 .

In Figs. 6 and 7, we examine the total average profit for different values of α and R_1 . The curve in Figure 6 becomes flatter as the α value decreases though for Fig. 7, the variation in total average profit is less significant with the change in number of installments.

8 | Concluding Remarks

This study follows the assumptions of Soni and Joshi [7] work and extends their work by incorporating the concept of installments. Here the supplier offers a partial trade credit to the retailer and the retailer offers a partial trade credit to its customer. After fulfilling the down payment, the customer pays the rest of the price by R_1 number of installments to the retailer.

Based on the above situation, we investigate the retailer's inventory model as a profit maximization problem. We aim to determine the retailer's optimal policy under the cases $N \leq M$ and $N \geq M$ respectively. When the number of installmentss is one, this model reduces to Soni and Joshi [7] model. We have also presented a comparative study among the models under; 1) sales purchase system, 2) installment purchase system, and 3) delay-in-payment system. It is observed that the retailer can gain maximum profit with installment purchase system than delay-in-payment. Also, this study analyzes that with more down payment the average profit increases, therefore 100% down payment i.e. sales purchase system gives the best average profit among the three purchase systems though it is sometimes impractical for the consumers. Furthermore, numerical examples and graphical illustrations are given to illustrate the proposed model. Based on the computational results, the following managerial insights can be drawn:

- I. In any scenario sales purchase system is always better because the retailer is receiving the payment instantaneously at the time of selling the product.
- II. In "buy-now-pay-later" scenario, the partial trade credit with installment policy is better than without installment policy as the retailer is receiving some amount gradually and its interest too.
- III. Compared with the other purchase systems, installment makes it easier for the customer to purchase within their budget and the retailer is also gaining by it.
- IV. More down payment may lead to higher profit for the business manager.
- V. A smaller number of installments leads to more profit for the business manager.

The proposed model can be extended in several ways. It can be applied to a supply chain model or we could generalize this with shortage, lost sales, substitution, etc. Also, the deterioration rate and the demand function can be dependent on various other parameters such as selling price, advertisement etc.

Data Availability Statement

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

Conflict of Interest

The authors declare that they have no conflict regarding the publication of this article.

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