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A Revised Inventory Model with Shortages in Fuzzy and Intuitionistic Fuzzy Frameworks under Two Warehouses

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Abstract

This study considers a two-warehouse inventory system with quadratic demand, different deterioration rates and fully backlogged shortages. Ordering, holding, deterioration and shortage costs are represented by crisp values, triangular fuzzy numbers or triangular intuitionistic fuzzy numbers. The warehouse balance determines the time at which the owned stock is exhausted, and signed-distance rankings give scalar average-cost objectives for the two fuzzy formulations. Numerical optimization determines the rented-warehouse release time, the stock-exhaustion time, the replenishment cycle and the order quantity. A worked example and one-parameter sensitivity calculations compare the three cost assessments. The results show how the chosen cost spreads change the computed schedule and average cost; the comparisons concern these assessments rather than measured operating savings.

Keywords: Inventory, Two warehouses, Fuzzy sets, Intuitionistic fuzzy sets, Quadratic demand, Fuzzy total cost, Intuitionistic fuzzy total cost.

1| Introduction

Inventory planning must balance ordering, storage, deterioration and shortage costs. For seasonal goods, demand can change during a replenishment cycle, while cost estimates may be imprecise. Fuzzy sets [1] and intuitionistic fuzzy sets [2] provide ways to represent such assessments; general foundations are given by Zimmermann [3]. Fuzzy inventory models have considered unreliable production, uncertain order quantities, fuzzy demand and alternative cost rankings [4-13].

Inventory policies also depend on the treatment of demand and lead time. Fuzzy mixture and continuous-review models combine these features in different ways [14-16]. When an owned warehouse has limited capacity, a rented warehouse can accommodate additional stock. Two-storage models examine the resulting release and replenishment policies [17], [18]. Inflation, credit periods and variable holding costs provide further extensions [19-21].

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Recent formulations include partial backlogging, ramp demand, fuzzy warehouse costs and finite product lifetimes [22-27]. Defuzzification methods compare numerical rankings of imprecise inputs [28].

Malik and Garg [29] study a fuzzy two-warehouse model with linear demand and no shortages. The present formulation considers quadratic demand, fully backlogged shortages and both triangular fuzzy and triangular intuitionistic fuzzy costs. These changes are evaluated through the same physical inventory balance in all three cost settings.

The aim is to determine a replenishment schedule and its average cost for specified demand and cost assessments. The signed-distance rankings convert the uncertain cost coefficients into scalar coefficients; the inventory trajectories themselves remain deterministic. Section 2 gives the required definitions, Section 3 develops the three formulations, and Sections 4 and 5 present numerical solutions and sensitivity calculations. Section 6 concludes the paper.

2| Preliminaries

Some basic definitions related to fuzzy set theory and intuitionistic fuzzy set theory are provided in this section.

2.1| Fuzzy Set

Let X be a universal set whose elements are denoted by x. A fuzzy set $\tilde{A}$ defined on X consists of ordered pairs:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X,\ 0 \leq \mu_{\tilde{A}}(x) \leq 1\},$$

where $\mu_{\tilde{A}}(x)$ represents the degree of membership of element x in the fuzzy set $\tilde{A}$.

2.2| Intuitionistic Fuzzy Set

Consider a non-empty set X. Let $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ be two functions representing the membership and non-membership degrees, respectively. An intuitionistic fuzzy set A on X is defined as

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\},$$

subject to the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in X$.

For two intuitionistic fuzzy sets A and B on X, A is said to be contained in B, denoted by $A \subseteq B$, if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$.

2.3| Triangular Fuzzy Number

A fuzzy number $\tilde{A}$ is called a triangular fuzzy number, denoted by $\tilde{A} = (a_1, a_2, a_3)$ with $a_1 < a_2 < a_3$, if its membership function $\mu_{\tilde{A}}(x)$ is given by:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases}$$

where a_1, a_2 and a_3 are respectively the lower, modal and upper values of $\tilde{A}$.

2.4| Intuitionistic Triangular Fuzzy Number

An intuitionistic fuzzy set $\tilde{A}$ on $\mathbb{R}$ is called an intuitionistic triangular fuzzy number, denoted by $\tilde{A} = \langle (a_1, a_2, a_3); (a_1', a_2, a_3')\rangle$, where $a_1' \leq a_1 < a_2 < a_3 \leq a_3'$.

The membership function $\mu_{\tilde{A}}(x)$ and non-membership function $\nu_{\tilde{A}}(x)$ are defined as:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases}$$

$$\nu_{\tilde{A}}(x) = \begin{cases} \dfrac{a_2 - x}{a_2 - a_1'}, & a_1' \le x \le a_2 \\ \dfrac{x - a_2}{a_3' - a_2}, & a_2 \le x \le a_3' \\ 1, & \text{otherwise} \end{cases}$$

with the condition $0 \le \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \le 1 \ \forall x \in \mathbb{R}$. Here a_2 is the modal value, (a_1, a_3) is the membership support and (a_1', a_3') is the non-membership support.

2.5| Signed Distance Method for Triangular Fuzzy Number (TFN)

Let $\tilde{A} = (a_1, a_2, a_3)$ be a triangular fuzzy number with $a_1 < a_2 < a_3$. The signed distance of $\tilde{A}$ from $\tilde{0} = (0, 0, 0)$ is defined as:

$$d(\tilde{A}, \tilde{0}) = \frac{1}{4}(a_1 + 2a_2 + a_3).$$

This value $d(\tilde{A}, \tilde{0})$ is called the defuzzified crisp value of $\tilde{A}$.

2.6| Signed Distance Method for Triangular Intuitionistic Fuzzy Number (TIFN)

Let $\tilde{A} = \langle (a_1, a_2, a_3); (a_1', a_2, a_3') \rangle$ be an intuitionistic triangular fuzzy number, where $a_1' \le a_1 < a_2 < a_3 \le a_3'$.

For the intuitionistic number, we use the average of the signed-distance ranks of its membership and non-membership triangles:

$$d(\tilde{A}, \tilde{0}) = \frac{1}{8}(a_1 + a_1' + 4a_2 + a_3 + a_3').$$

This crisp value $d(\tilde{A}, \tilde{0})$ is used as the defuzzified value of $\tilde{A}$.

3| Model Development

3.1| Crisp Model

3.1.1| Assumption

(i) The proposed inventory model considers a single product. (ii) The demand of

the product is $a + bt + ct^2$.

(iii) The lead time is considered to be zero, and shortages are allowed.

(iv) The proposed model considers two warehouses: owned and rented warehouses.

3.1.2| Notation

α_R: Deterioration rate in the rented warehouse.

α_W: Deterioration rate in the owned warehouse, $\alpha_W > \alpha_R$.

β_1: Holding cost per item per unit time in the rented warehouse.

β_2: Holding cost per item per unit time in the owned warehouse.

A_0: Ordering cost per order.

d_c: Cost per deteriorated item.

C_s: Shortage cost per item per unit time.

T: Replenishment cycle length.

TIC: Total inventory cost.

W_0: Maximum inventory level in the own warehouse.

T^*: Optimal Time for a cycle.

TIC^*: Minimum total inventory cost.

$d_F(\widetilde{TIC})$: Defuzzified total inventory cost.

$d_F(\widetilde{TIC}^I)$: Defuzzified total inventory cost for Intuitionistic Model.

$d_F(\widetilde{TIC})^*$: Minimum Defuzzified total inventory cost.

$d_F(\widetilde{TIC}^I)^*$: Minimum Defuzzified total inventory cost for Intuitionistic Model.

3.1.3| Model Formulation. In this proposed model, we consider an inventory system under a two-warehouse system for seasonal products.

For the rented warehouse, the inventory level $I_R(t)$ reaches zero at time t_1. During the period $(0, t_1)$, the demand of the customers is fulfilled from the Rented Warehouse. Thus, during this period, some products deteriorate in the own Warehouse; $I_{W1}(t)$ represents the inventory level of such warehouse. After the rented warehouse becomes empty, the customers' demand is fulfilled by the own warehouse. During the time period (t_1, t_2) where $t_2 = t_1 + t_1'$, $I_{W2}(t)$ also represents the inventory level. After this time period, shortages are created at the same customer demand rate per unit time during $[t_2, T]$, where $T = t_2 + t_2'$ and $I_{W3}(t)$ represents the inventory level during this period.

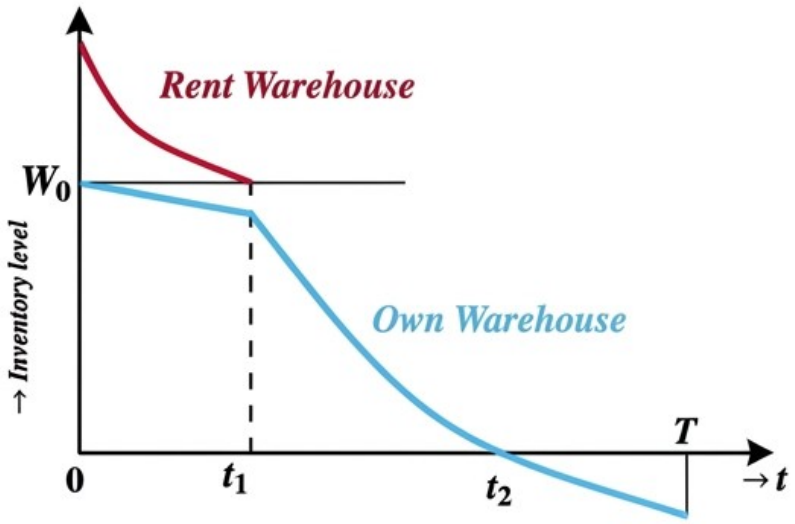


Figure 1. Inventory levels during a replenishment cycle.

$$\frac{dI_R(t)}{dt} + \alpha_R I_R(t) = -(a + bt + ct^2), \quad 0 \le t \le t_1 \tag{1}$$

$$I_R(t_1) = 0$$

$$\frac{dI_{W1}(t)}{dt} + \alpha_W I_{W1}(t) = 0, \quad 0 \le t \le t_1 \tag{2}$$

$$I_{W1}(0) = W_0$$

$$\frac{dI_{W2}(t)}{dt} + \alpha_W I_{W2}(t) = -(a + bt + ct^2), \quad t_1 \le t \le t_2 \tag{3}$$

$$I_{W2}(t_2) = 0$$

$$I_{W1}(t_1) = I_{W2}(t_1)$$

$$\frac{dI_{W3}(t)}{dt} = -(a + bt + ct^2), \quad t_2 \le t \le T \tag{4}$$

$$I_{W3}(t_2) = 0$$

Solving (1) we get

$$I_R(t) = \left[-\frac{a}{\alpha_R} - \frac{bt}{\alpha_R} + \frac{b}{\alpha_R^2} - \frac{ct^2}{\alpha_R} + \frac{2ct}{\alpha_R^2} - \frac{2c}{\alpha_R^3}\right] + \left[\frac{a}{\alpha_R} + \frac{bt_1}{\alpha_R} - \frac{b}{\alpha_R^2} + \frac{ct_1^2}{\alpha_R} - \frac{2ct_1}{\alpha_R^2} + \frac{2c}{\alpha_R^3}\right] e^{\alpha_R(t_1-t)}$$

Solving (2) we get

$$I_{W1}(t) = W_0 e^{-\alpha_W t}$$

Solving (3) we get

$$I_{W2}(t) = \left[-\frac{a}{\alpha_W} - \frac{bt}{\alpha_W} + \frac{b}{\alpha_W^2} - \frac{ct^2}{\alpha_W} + \frac{2ct}{\alpha_W^2} - \frac{2c}{\alpha_W^3}\right] + \left[\frac{a}{\alpha_W} + \frac{bt_2}{\alpha_W} - \frac{b}{\alpha_W^2} + \frac{ct_2^2}{\alpha_W} - \frac{2ct_2}{\alpha_W^2} + \frac{2c}{\alpha_W^3}\right] e^{-\alpha_W(t-t_2)}$$

From (4) we get

$$I_{W3}(t) = \left[-at - \frac{bt^2}{2} - \frac{ct^3}{3}\right] + at_2 + \frac{bt_2^2}{2} + \frac{ct_2^3}{3}$$

From $I_{W1}(t_1) = I_{W2}(t_1)$ we get

$$W_0 e^{-\alpha_W t_1} = \left[-\frac{a}{\alpha_W} - \frac{bt_1}{\alpha_W} + \frac{b}{\alpha_W^2} - \frac{ct_1^2}{\alpha_W} + \frac{2ct_1}{\alpha_W^2} - \frac{2c}{\alpha_W^3}\right] + \left[\frac{a}{\alpha_W} + \frac{bt_2}{\alpha_W} - \frac{b}{\alpha_W^2} + \frac{ct_2^2}{\alpha_W} - \frac{2ct_2}{\alpha_W^2} + \frac{2c}{\alpha_W^3}\right] e^{\alpha_W(t_2-t_1)}$$

$$\Rightarrow \left[\frac{a}{\alpha_W} + \frac{bt_2}{\alpha_W} - \frac{b}{\alpha_W^2} + \frac{ct_2^2}{\alpha_W} - \frac{2ct_2}{\alpha_W^2} + \frac{2c}{\alpha_W^3}\right]\left(\frac{e^{\alpha_W(t_2-t_1)}}{\alpha_W} - \frac{1}{\alpha_W}\right) = \frac{W_0 e^{-\alpha_W t_1}}{\alpha_W} - \frac{b}{\alpha_W^2}(t_2-t_1) - \frac{c}{\alpha_W^2}(t_2^2-t_1^2) + \frac{2c}{\alpha_W^3}(t_2-t_1) \tag{5}$$

Inventory ordering cost, IOC $= A_0$.

And the Inventory storage cost in the rented warehouse is

$$\begin{aligned}\mathrm{ISC}_R &= \beta_1 \int_0^{t_1} I_R(t)\,dt \\ &= \beta_1 \left[\left(-\frac{at_1}{\alpha_R} - \frac{bt_1^2}{2\alpha_R} - \frac{ct_1^3}{3\alpha_R} - \frac{a}{\alpha_R^2} + \frac{b}{\alpha_R^3} - \frac{2c}{\alpha_R^4}\right) + \left(\frac{a}{\alpha_R} + \frac{bt_1}{\alpha_R} - \frac{b}{\alpha_R^2} + \frac{ct_1^2}{\alpha_R} - \frac{2ct_1}{\alpha_R^2} + \frac{2c}{\alpha_R^3}\right)\frac{e^{\alpha_R t_1}}{\alpha_R}\right]\end{aligned}$$

Similarly, the Inventory deterioration cost in a rented warehouse is

$$\begin{aligned}\mathrm{IDC}_R &= d_c \int_0^{t_1} \alpha_R I_R(t)\,dt \\ &= d_c \alpha_R \left[\left(-\frac{at_1}{\alpha_R} - \frac{bt_1^2}{2\alpha_R} - \frac{ct_1^3}{3\alpha_R} - \frac{a}{\alpha_R^2} + \frac{b}{\alpha_R^3} - \frac{2c}{\alpha_R^4}\right) + \left(\frac{a}{\alpha_R} + \frac{bt_1}{\alpha_R} - \frac{b}{\alpha_R^2} + \frac{ct_1^2}{\alpha_R} - \frac{2ct_1}{\alpha_R^2} + \frac{2c}{\alpha_R^3}\right)\frac{e^{\alpha_R t_1}}{\alpha_R}\right]\end{aligned}$$

The Inventory storage cost in its own warehouse is

$$\begin{aligned}\mathrm{ISC}_O &= \beta_2 \left[\int_0^{t_1} I_{W1}(t)\,dt + \int_{t_1}^{t_2} I_{W2}(t)\,dt\right] \\ &= \beta_2 \left[\frac{W_0}{\alpha_W}\left(1 - e^{-\alpha_W t_1}\right) + \left\{-\frac{a}{\alpha_W}(t_2-t_1) - \frac{b}{2\alpha_W}(t_2^2-t_1^2) + \frac{b}{\alpha_W^2}(t_2-t_1) - \frac{c}{3\alpha_W}(t_2^3-t_1^3) + \frac{c}{\alpha_W^2}(t_2^2-t_1^2) - \frac{2c}{\alpha_W^3}(t_2-t_1)\right\} + \left(\frac{a}{\alpha_W} + \frac{bt_2}{\alpha_W} - \frac{b}{\alpha_W^2} + \frac{ct_2^2}{\alpha_W} - \frac{2ct_2}{\alpha_W^2} + \frac{2c}{\alpha_W^3}\right)\left(\frac{e^{\alpha_W(t_2-t_1)} - 1}{\alpha_W}\right)\right]\end{aligned}$$

And Inventory deterioration Cost in own warehouse is

$$\begin{aligned}\mathrm{IDC}_O &= d_c\left[\int_0^{t_1}\alpha_W I_{W1}(t)\,dt+\int_{t_1}^{t_2}\alpha_W I_{W2}(t)\,dt\right]\\ &= d_c\Bigg[-W_0e^{-\alpha_W t_1}+W_0\\ &\quad+\left\{-a(t_2-t_1)-\frac{b}{2}(t_2^2-t_1^2)+\frac{b}{\alpha_W}(t_2-t_1)-\frac{c}{3}(t_2^3-t_1^3)+\frac{c}{\alpha_W}(t_2^2-t_1^2)-\frac{2c}{\alpha_W^2}(t_2-t_1)\right\}\\ &\quad+\left(a+bt_2-\frac{b}{\alpha_W}+ct_2^2-\frac{2ct_2}{\alpha_W}+\frac{2c}{\alpha_W^2}\right)\left(\frac{e^{\alpha_W(t_2-t_1)}}{\alpha_W}-\frac{1}{\alpha_W}\right)\Bigg]\end{aligned}$$

Inventory shortage cost,

$$\begin{aligned}I_{\text{Shortage}} &= -C_s\int_{t_2}^{T} I_{W3}(t)\,dt\\ &= C_s\left[aT\left(\frac{T}{2}-t_2\right)+\frac{bT}{2}\left(\frac{T^2}{3}-t_2^2\right)+\frac{cT}{3}\left(\frac{T^3}{4}-t_2^3\right)+\frac{at_2^2}{2}+\frac{bt_2^3}{3}+\frac{ct_2^4}{4}\right]\end{aligned}$$

Therefore, the Total Inventory Cost, TIC is

$$\mathrm{TIC}=\frac{1}{T}\left[\mathrm{IOC}+\mathrm{ISC}_R+\mathrm{ISC}_O+\mathrm{IDC}_R+\mathrm{IDC}_O+I_{\text{Shortage}}\right]$$

$$\begin{aligned}\mathrm{TIC} &= \frac{1}{T}\Bigg[A_0+(\beta_1+d_c\alpha_R)\Bigg\{\left(-\frac{at_1}{\alpha_R}-\frac{bt_1^2}{2\alpha_R}-\frac{ct_1^3}{3\alpha_R}-\frac{a}{\alpha_R^2}+\frac{b}{\alpha_R^3}-\frac{2c}{\alpha_R^4}\right)\\ &\quad+\left(\frac{a}{\alpha_R}+\frac{bt_1}{\alpha_R}-\frac{b}{\alpha_R^2}+\frac{ct_1^2}{\alpha_R}-\frac{2ct_1}{\alpha_R^2}+\frac{2c}{\alpha_R^3}\right)\frac{e^{\alpha_R t_1}}{\alpha_R}\Bigg\}+(\beta_2+d_c\alpha_W)\Bigg\{\left(\frac{W_0}{\alpha_W}-W_0\frac{e^{-\alpha_W t_1}}{\alpha_W}\right)\\ &\quad+\Bigg\{-\frac{a}{\alpha_W}(t_2-t_1)-\frac{b}{2\alpha_W}(t_2^2-t_1^2)+\frac{b}{\alpha_W^2}(t_2-t_1)-\frac{c}{3\alpha_W}(t_2^3-t_1^3)+\frac{c}{\alpha_W^2}(t_2^2-t_1^2)\\ &\qquad-\frac{2c}{\alpha_W^3}(t_2-t_1)\Bigg\}+\left(\frac{a}{\alpha_W}+\frac{bt_2}{\alpha_W}-\frac{b}{\alpha_W^2}+\frac{ct_2^2}{\alpha_W}-\frac{2ct_2}{\alpha_W^2}+\frac{2c}{\alpha_W^3}\right)\left(\frac{e^{\alpha_W(t_2-t_1)}}{\alpha_W}-\frac{1}{\alpha_W}\right)\Bigg\}\\ &\quad+C_s\left\{aT\left(\frac{T}{2}-t_2\right)+\frac{bT}{2}\left(\frac{T^2}{3}-t_2^2\right)+\frac{cT}{3}\left(\frac{T^3}{4}-t_2^3\right)+\frac{at_2^2}{2}+\frac{bt_2^3}{3}+\frac{ct_2^4}{4}\right\}\Bigg]\end{aligned}$$

using (5) we get

$$\begin{aligned}\mathrm{TIC} &= \frac{1}{T}\Bigg[A_0+(\beta_1+d_c\alpha_R)\Bigg\{\left(-\frac{at_1}{\alpha_R}-\frac{bt_1^2}{2\alpha_R}-\frac{ct_1^3}{3\alpha_R}-\frac{a}{\alpha_R^2}+\frac{b}{\alpha_R^3}-\frac{2c}{\alpha_R^4}\right)\\ &\quad+\left(\frac{a}{\alpha_R}+\frac{bt_1}{\alpha_R}-\frac{b}{\alpha_R^2}+\frac{ct_1^2}{\alpha_R}-\frac{2ct_1}{\alpha_R^2}+\frac{2c}{\alpha_R^3}\right)\frac{e^{\alpha_R t_1}}{\alpha_R}\Bigg\}+(\beta_2+d_c\alpha_W)\Bigg\{\frac{W_0}{\alpha_W}-\frac{b}{2\alpha_W}(t_2^2-t_1^2)-\frac{c}{3\alpha_W}(t_2^3-t_1^3)\\ &\quad-\frac{a}{\alpha_W}(t_2-t_1)\Bigg\}+C_s\left\{aT\left(\frac{T}{2}-t_2\right)+\frac{bT}{2}\left(\frac{T^2}{3}-t_2^2\right)+\frac{cT}{3}\left(\frac{T^3}{4}-t_2^3\right)+\frac{at_2^2}{2}+\frac{bt_2^3}{3}+\frac{ct_2^4}{4}\right\}\Bigg]\end{aligned}\tag{6}$$

The feasible schedules satisfy $0\le t_1<t_2\le T$ and the warehouse matching condition. Multiplying the owned-warehouse differential equation by its integrating factor gives the useful equivalent condition

$$\int_{t_1}^{t_2} e^{\alpha_W u}(a+bu+cu^2)\,du=W_0.$$

Demand is measured from the beginning of each cycle, with $a > 0$, $b, c \geq 0$ and positive deterioration rates. Shortages are fully backlogged and cleared at the next instantaneous replenishment. If $D(u) = a + bu + cu^2$, the order quantity is

$$Q = I_R(0) + W_0 + \int_{t_2}^{T} D(u)\, du.$$

The first two terms restore both warehouses and the last term clears the accumulated backlog. The objective is to minimize the average cost in (6) over feasible schedules. The warehouse equality determines a unique t_2 for each t_1 when $c > 0$ or $b > 0$ or $a > 0$. Numerical calculations below enforce this equality explicitly.

3.2| Fuzzy Model

In this section, we have presented our fuzzy inventory model through the 'Signed distance method'. As for seasonal products and the global market scenario, the price of products fluctuates. With ordering, storage, and deterioration costs, shortage cost that means, these cost constraints cannot be assumed fixed. To avoid this type of problem, we consider these costs for both warehouses as a triangular fuzzy number. To develop this inventory model, we assume the following triangular fuzzy numbers for ordering cost, deterioration cost, storage cost, and shortage costs.

(1) $\widetilde{A}_0 = (A_0 - \Delta_1, A_0, A_0 + \Delta_2)$ where $0 < \Delta_1 < A_0$ and $\Delta_2 > 0$
(2) $\widetilde{d}_c = (d_c - \Delta_3, d_c, d_c + \Delta_4)$ where $0 < \Delta_3 < d_c$ and $\Delta_4 > 0$
(3) $\widetilde{\beta}_1 = (\beta_1 - \Delta_5, \beta_1, \beta_1 + \Delta_6)$ where $0 < \Delta_5 < \beta_1$ and $\Delta_6 > 0$
(4) $\widetilde{\beta}_2 = (\beta_2 - \Delta_7, \beta_2, \beta_2 + \Delta_8)$ where $0 < \Delta_7 < \beta_2$ and $\Delta_8 > 0$
(5) $\widetilde{C}_s = (C_s - \Delta_9, C_s, C_s + \Delta_{10})$ where $0 < \Delta_9 < C_s$ and $\Delta_{10} > 0$

using 2.5 we get,

1. $d(\widetilde{A}_0, 0) = A_0 + \frac{1}{4}(\Delta_2 - \Delta_1)$
2. $d(\widetilde{d}_c, 0) = d_c + \frac{1}{4}(\Delta_4 - \Delta_3)$
3. $d(\widetilde{\beta}_1, 0) = \beta_1 + \frac{1}{4}(\Delta_6 - \Delta_5)$
4. $d(\widetilde{\beta}_2, 0) = \beta_2 + \frac{1}{4}(\Delta_8 - \Delta_7)$
5. $d(\widetilde{C}_s, 0) = C_s + \frac{1}{4}(\Delta_{10} - \Delta_9)$

We can transform the crisp model of the total cost into the fuzzy environment. From the equation (6) we can deduce that

$$\widetilde{TIC} = (\widetilde{TIC}_1, \widetilde{TIC}_2, \widetilde{TIC}_3)$$

$$\begin{aligned}\widetilde{TIC}_1 = \frac{1}{T}\Bigg[&(A_0 - \Delta_1) + \{(\beta_1 - \Delta_5) + (d_c - \Delta_3)\alpha_R\}\left\{\left(-\frac{at_1}{\alpha_R} - \frac{bt_1^2}{2\alpha_R} - \frac{ct_1^3}{3\alpha_R} - \frac{a}{\alpha_R^2} + \frac{b}{\alpha_R^3} - \frac{2c}{\alpha_R^4}\right)\right.\\ &\left. + \left(\frac{a}{\alpha_R} + \frac{bt_1}{\alpha_R} - \frac{b}{\alpha_R^2} + \frac{ct_1^2}{\alpha_R} - \frac{2ct_1}{\alpha_R^2} + \frac{2c}{\alpha_R^3}\right)\frac{e^{t_1\alpha_R}}{\alpha_R}\right\}\\ &+ \{(\beta_2 - \Delta_7) + (d_c - \Delta_3)\alpha_W\}\left\{\frac{W_0}{\alpha_W} - \frac{a}{\alpha_W}(t_2 - t_1) - \frac{b}{2\alpha_W}(t_2^2 - t_1^2) - \frac{c}{3\alpha_W}(t_2^3 - t_1^3)\right\}\\ &+ (C_s - \Delta_9)\left\{aT\left(\frac{T}{2} - t_2\right) + \frac{bT}{2}\left(\frac{T^2}{3} - t_2^2\right) + \frac{cT}{3}\left(\frac{T^3}{4} - t_2^3\right) + \frac{at_2^2}{2} + \frac{bt_2^3}{3} + \frac{ct_2^4}{4}\right\}\Bigg]\end{aligned}$$

$$\widetilde{TIC}_2 = TIC$$

$$\widetilde{TIC}_3 = \frac{1}{T}\Bigg[(A_0+\Delta_2)+\{(\beta_1+\Delta_6)+(d_c+\Delta_4)\alpha_R\}\left\{\left(-\frac{at_1}{\alpha_R}-\frac{bt_1^2}{2\alpha_R}-\frac{ct_1^3}{3\alpha_R}-\frac{a}{\alpha_R^2}+\frac{b}{\alpha_R^3}-\frac{2c}{\alpha_R^4}\right)+\left(\frac{a}{\alpha_R}+\frac{bt_1}{\alpha_R}-\frac{b}{\alpha_R^2}+\frac{ct_1^2}{\alpha_R}-\frac{2ct_1}{\alpha_R^2}+\frac{2c}{\alpha_R^3}\right)\frac{e^{t_1\alpha_R}}{\alpha_R}\right\}$$
$$+\{(\beta_2+\Delta_8)+(d_c+\Delta_4)\alpha_W\}\left\{\frac{W_0}{\alpha_W}-\frac{a}{\alpha_W}(t_2-t_1)-\frac{b}{2\alpha_W}(t_2^2-t_1^2)-\frac{c}{3\alpha_W}(t_2^3-t_1^3)\right\}$$
$$+(C_s+\Delta_{10})\left\{aT\left(\frac{T}{2}-t_2\right)+\frac{bT}{2}\left(\frac{T^2}{3}-t_2^2\right)+\frac{cT}{3}\left(\frac{T^3}{4}-t_2^3\right)+\frac{at_2^2}{2}+\frac{bt_2^3}{3}+\frac{ct_2^4}{4}\right\}\Bigg]$$

using 2.5 we get,

$$d_F(\widetilde{TIC}) = \frac{\widetilde{TIC}_1 + 2\widetilde{TIC}_2 + \widetilde{TIC}_3}{4}$$

$$= \frac{1}{4T}\Bigg[(4A_0+\Delta_2-\Delta_1)+\{4\beta_1+(\Delta_6-\Delta_5)+(4d_c+\Delta_4-\Delta_3)\alpha_R\}\left\{\left(-\frac{at_1}{\alpha_R}-\frac{bt_1^2}{2\alpha_R}-\frac{ct_1^3}{3\alpha_R}-\frac{a}{\alpha_R^2}+\frac{b}{\alpha_R^3}-\frac{2c}{\alpha_R^4}\right)+\left(\frac{a}{\alpha_R}+\frac{bt_1}{\alpha_R}-\frac{b}{\alpha_R^2}+\frac{ct_1^2}{\alpha_R}-\frac{2ct_1}{\alpha_R^2}+\frac{2c}{\alpha_R^3}\right)\frac{e^{t_1\alpha_R}}{\alpha_R}\right\}$$
$$+\{(4\beta_2+\Delta_8-\Delta_7)+(4d_c+\Delta_4-\Delta_3)\alpha_W\}\left\{\frac{W_0}{\alpha_W}-\frac{a}{\alpha_W}(t_2-t_1)-\frac{b}{2\alpha_W}(t_2^2-t_1^2)-\frac{c}{3\alpha_W}(t_2^3-t_1^3)\right\}$$
$$+(4C_s+\Delta_{10}-\Delta_9)\left\{aT\left(\frac{T}{2}-t_2\right)+\frac{bT}{2}\left(\frac{T^2}{3}-t_2^2\right)+\frac{cT}{3}\left(\frac{T^3}{4}-t_2^3\right)+\frac{at_2^2}{2}+\frac{bt_2^3}{3}+\frac{ct_2^4}{4}\right\}\Bigg]$$

3.3| Intuitionistic Fuzzy Inventory Model

(1) $\widetilde{A_0}^I = (A_0-\Delta_1, A_0, A_0+\Delta_2; A_0-\Delta_1', A_0, A_0+\Delta_2')$ where $0<\Delta_1,\Delta_1'<A_0$ and $\Delta_2>0,\Delta_2'>0$.

(2) $\widetilde{d_c}^I = (d_c-\Delta_3, d_c, d_c+\Delta_4; d_c-\Delta_3', d_c, d_c+\Delta_4')$ where $0<\Delta_3,\Delta_3'<d_c$ and $\Delta_4>0,\Delta_4'>0$.

(3) $\widetilde{\beta_1}^I = (\beta_1-\Delta_5, \beta_1, \beta_1+\Delta_6; \beta_1-\Delta_5', \beta_1, \beta_1+\Delta_6')$ where $0<\Delta_5,\Delta_5'<\beta_1$ and $\Delta_6>0,\Delta_6'>0$.

(4) $\widetilde{\beta_2}^I = (\beta_2-\Delta_7, \beta_2, \beta_2+\Delta_8; \beta_2-\Delta_7', \beta_2, \beta_2+\Delta_8')$ where $0<\Delta_7,\Delta_7'<\beta_2$ and $\Delta_8>0,\Delta_8'>0$.

(5) $\widetilde{C_s}^I = (C_s-\Delta_9, C_s, C_s+\Delta_{10}; C_s-\Delta_9', C_s, C_s+\Delta_{10}')$ where $0<\Delta_9,\Delta_9'<C_s$ and $\Delta_{10}>0,\Delta_{10}'>0$.

For each coefficient, the primed left and right spreads are at least as large as the corresponding unprimed spreads. This nesting ensures that the membership and non-membership functions satisfy the intuitionistic fuzzy condition. We transform the crisp model of the total cost into an intuitionistic fuzzy environment. From the equation (6)

$$\widetilde{TIC}^I = (\widetilde{TIC}_1^I, \widetilde{TIC}_2^I, \widetilde{TIC}_3^I; \widetilde{TIC'}_1^I, \widetilde{TIC'}_2^I, \widetilde{TIC'}_3^I).$$

$$\widetilde{TIC}_1^I = \frac{1}{T}\Bigg[(A_0-\Delta_1)+\{(\beta_1-\Delta_5)+(d_c-\Delta_3)\alpha_R\}\left\{\left(-\frac{at_1}{\alpha_R}-\frac{bt_1^2}{2\alpha_R}-\frac{ct_1^3}{3\alpha_R}-\frac{a}{\alpha_R^2}+\frac{b}{\alpha_R^3}-\frac{2c}{\alpha_R^4}\right)+\left(\frac{a}{\alpha_R}+\frac{bt_1}{\alpha_R}-\frac{b}{\alpha_R^2}+\frac{ct_1^2}{\alpha_R}-\frac{2ct_1}{\alpha_R^2}+\frac{2c}{\alpha_R^3}\right)\frac{e^{t_1\alpha_R}}{\alpha_R}\right\}$$
$$+\{(\beta_2-\Delta_7)+(d_c-\Delta_3)\alpha_W\}\left\{\frac{W_0}{\alpha_W}-\frac{a}{\alpha_W}(t_2-t_1)-\frac{b}{2\alpha_W}(t_2^2-t_1^2)-\frac{c}{3\alpha_W}(t_2^3-t_1^3)\right\}$$
$$+(C_s-\Delta_9)\left\{aT\left(\frac{T}{2}-t_2\right)+\frac{bT}{2}\left(\frac{T^2}{3}-t_2^2\right)+\frac{cT}{3}\left(\frac{T^3}{4}-t_2^3\right)+\frac{at_2^2}{2}+\frac{bt_2^3}{3}+\frac{ct_2^4}{4}\right\}\Bigg]$$

$$\widetilde{TIC}_2^I = TIC$$

$$\widetilde{TIC}_3^I = \frac{1}{T}\Bigg[(A_0+\Delta_2)+\{(\beta_1+\Delta_6)+(d_c+\Delta_4)\alpha_R\}\left\{\left(-\frac{at_1}{\alpha_R}-\frac{bt_1^2}{2\alpha_R}-\frac{ct_1^3}{3\alpha_R}-\frac{a}{\alpha_R^2}+\frac{b}{\alpha_R^3}-\frac{2c}{\alpha_R^4}\right)+\left(\frac{a}{\alpha_R}+\frac{bt_1}{\alpha_R}-\frac{b}{\alpha_R^2}+\frac{ct_1^2}{\alpha_R}-\frac{2ct_1}{\alpha_R^2}+\frac{2c}{\alpha_R^3}\right)\frac{e^{t_1\alpha_R}}{\alpha_R}\right\}+\{(\beta_2+\Delta_8)+(d_c+\Delta_4)\alpha_W\}\left\{\frac{W_0}{\alpha_W}-\frac{a}{\alpha_W}(t_2-t_1)-\frac{b}{2\alpha_W}(t_2^2-t_1^2)-\frac{c}{3\alpha_W}(t_2^3-t_1^3)\right\}+(C_s+\Delta_{10})\left\{aT\left(\frac{T}{2}-t_2\right)+\frac{bT}{2}\left(\frac{T^2}{3}-t_2^2\right)+\frac{cT}{3}\left(\frac{T^3}{4}-t_2^3\right)+\frac{at_2^2}{2}+\frac{bt_2^3}{3}+\frac{ct_2^4}{4}\right\}\Bigg]$$

$$\widetilde{TIC'}_1^I = \frac{1}{T}\Bigg[(A_0-\Delta_1')+\{(\beta_1-\Delta_5')+(d_c-\Delta_3')\alpha_R\}\left\{\left(-\frac{at_1}{\alpha_R}-\frac{bt_1^2}{2\alpha_R}-\frac{ct_1^3}{3\alpha_R}-\frac{a}{\alpha_R^2}+\frac{b}{\alpha_R^3}-\frac{2c}{\alpha_R^4}\right)+\left(\frac{a}{\alpha_R}+\frac{bt_1}{\alpha_R}-\frac{b}{\alpha_R^2}+\frac{ct_1^2}{\alpha_R}-\frac{2ct_1}{\alpha_R^2}+\frac{2c}{\alpha_R^3}\right)\frac{e^{t_1\alpha_R}}{\alpha_R}\right\}+\{(\beta_2-\Delta_7')+(d_c-\Delta_3')\alpha_W\}\left\{\frac{W_0}{\alpha_W}-\frac{a}{\alpha_W}(t_2-t_1)-\frac{b}{2\alpha_W}(t_2^2-t_1^2)-\frac{c}{3\alpha_W}(t_2^3-t_1^3)\right\}+(C_s-\Delta_9')\left\{aT\left(\frac{T}{2}-t_2\right)+\frac{bT}{2}\left(\frac{T^2}{3}-t_2^2\right)+\frac{cT}{3}\left(\frac{T^3}{4}-t_2^3\right)+\frac{at_2^2}{2}+\frac{bt_2^3}{3}+\frac{ct_2^4}{4}\right\}\Bigg]$$

$$\widetilde{TIC'}_2^I = TIC$$

$$\widetilde{TIC'}_3^I = \frac{1}{T}\Bigg[(A_0+\Delta_2')+\{(\beta_1+\Delta_6')+(d_c+\Delta_4')\alpha_R\}\left\{\left(-\frac{at_1}{\alpha_R}-\frac{bt_1^2}{2\alpha_R}-\frac{ct_1^3}{3\alpha_R}-\frac{a}{\alpha_R^2}+\frac{b}{\alpha_R^3}-\frac{2c}{\alpha_R^4}\right)+\left(\frac{a}{\alpha_R}+\frac{bt_1}{\alpha_R}-\frac{b}{\alpha_R^2}+\frac{ct_1^2}{\alpha_R}-\frac{2ct_1}{\alpha_R^2}+\frac{2c}{\alpha_R^3}\right)\frac{e^{t_1\alpha_R}}{\alpha_R}\right\}+\{(\beta_2+\Delta_8')+(d_c+\Delta_4')\alpha_W\}\left\{\frac{W_0}{\alpha_W}-\frac{a}{\alpha_W}(t_2-t_1)-\frac{b}{2\alpha_W}(t_2^2-t_1^2)-\frac{c}{3\alpha_W}(t_2^3-t_1^3)\right\}+(C_s+\Delta_{10}')\left\{aT\left(\frac{T}{2}-t_2\right)+\frac{bT}{2}\left(\frac{T^2}{3}-t_2^2\right)+\frac{cT}{3}\left(\frac{T^3}{4}-t_2^3\right)+\frac{at_2^2}{2}+\frac{bt_2^3}{3}+\frac{ct_2^4}{4}\right\}\Bigg]$$

Using Section 2.6,

$$d_F(\widetilde{TIC}^I) = \frac{1}{2}\left[\frac{\widetilde{TIC}_1^I+2\widetilde{TIC}_2^I+\widetilde{TIC}_3^I}{4}+\frac{\widetilde{TIC'}_1^I+2\widetilde{TIC'}_2^I+\widetilde{TIC'}_3^I}{4}\right]$$

$$
\begin{aligned}
= \frac{1}{8T}\Bigg[& (4A_0+\Delta_2-\Delta_1)+\{(4\beta_1+\Delta_6-\Delta_5)+(4d_c+\Delta_4-\Delta_3)\alpha_R\}\left\{\left(-\frac{at_1}{\alpha_R}-\frac{bt_1^2}{2\alpha_R}-\frac{ct_1^3}{3\alpha_R}-\frac{a}{\alpha_R^2}+\frac{b}{\alpha_R^3}-\frac{2c}{\alpha_R^4}\right)\right. \\
& \left.+\left(\frac{a}{\alpha_R}+\frac{bt_1}{\alpha_R}-\frac{b}{\alpha_R^2}+\frac{ct_1^2}{\alpha_R}-\frac{2ct_1}{\alpha_R^2}+\frac{2c}{\alpha_R^3}\right)\frac{e^{t_1\alpha_R}}{\alpha_R}\right\} \\
& +\{(4\beta_2+\Delta_8-\Delta_7)+(4d_c+\Delta_4-\Delta_3)\alpha_W\}\left\{\frac{W_0}{\alpha_W}-\frac{a}{\alpha_W}(t_2-t_1)-\frac{b}{2\alpha_W}(t_2^2-t_1^2)-\frac{c}{3\alpha_W}(t_2^3-t_1^3)\right\} \\
& +(4C_s+\Delta_{10}-\Delta_9)\left\{aT\left(\frac{T}{2}-t_2\right)+\frac{bT}{2}\left(\frac{T^2}{3}-t_2^2\right)+\frac{cT}{3}\left(\frac{T^3}{4}-t_2^3\right)+\frac{at_2^2}{2}+\frac{bt_2^3}{3}+\frac{ct_2^4}{4}\right\} \\
& +(4A_0+\Delta_2'-\Delta_1')+\{(4\beta_1+\Delta_6'-\Delta_5')+(4d_c+\Delta_4'-\Delta_3')\alpha_R\}\left\{\left(-\frac{at_1}{\alpha_R}-\frac{bt_1^2}{2\alpha_R}-\frac{ct_1^3}{3\alpha_R}-\frac{a}{\alpha_R^2}+\frac{b}{\alpha_R^3}-\frac{2c}{\alpha_R^4}\right)\right. \\
& \left.+\left(\frac{a}{\alpha_R}+\frac{bt_1}{\alpha_R}-\frac{b}{\alpha_R^2}+\frac{ct_1^2}{\alpha_R}-\frac{2ct_1}{\alpha_R^2}+\frac{2c}{\alpha_R^3}\right)\frac{e^{t_1\alpha_R}}{\alpha_R}\right\} \\
& +\{(4\beta_2+\Delta_8'-\Delta_7')+(4d_c+\Delta_4'-\Delta_3')\alpha_W\}\left\{\frac{W_0}{\alpha_W}-\frac{a}{\alpha_W}(t_2-t_1)-\frac{b}{2\alpha_W}(t_2^2-t_1^2)-\frac{c}{3\alpha_W}(t_2^3-t_1^3)\right\} \\
& +(4C_s+\Delta_{10}'-\Delta_9')\left\{aT\left(\frac{T}{2}-t_2\right)+\frac{bT}{2}\left(\frac{T^2}{3}-t_2^2\right)+\frac{cT}{3}\left(\frac{T^3}{4}-t_2^3\right)+\frac{at_2^2}{2}+\frac{bt_2^3}{3}+\frac{ct_2^4}{4}\right\}\Bigg]
\end{aligned}
$$

4| Numerical Results

The example uses $\alpha_R = 0.05$, $\alpha_W = 0.07$, $\beta_1 = 0.08$, $\beta_2 = 0.05$, $A_0 = 30000$, $d_c = 3$, $C_s = 5$ and $W_0 = 2000$, with $a = 30$, $b = 12$ and $c = 1$. These are illustrative inputs, not observations from a warehouse.

4.1| Cost Assessments and Computed Schedules

For triangular fuzzy costs, the spread pairs $(\Delta_{2j-1}, \Delta_{2j})$ for ordering, deterioration, rented holding, owned holding and shortage costs are, respectively,

$$(100, 120),\ (0.05, 0.02),\ (0.004, 0.002),\ (0.001, 0.002),\ (0.1, 0.2).$$

The intuitionistic assessment uses these membership spreads and the following primed spread pairs:

$$(110, 135),\ (0.06, 0.08),\ (0.005, 0.006),\ (0.003, 0.005),\ (0.12, 0.6).$$

Applying the rankings in Sections 2.5 and 2.6 gives the scalar coefficients in Table 1. In particular, the intuitionistic shortage coefficient is $5 + (0.2 - 0.1 + 0.6 - 0.12)/8 = 5.0725$.

Table 1. Effective coefficients used in every numerical calculation.

Assessment	A_0	d_c	β_1	β_2	C_s
Crisp	30000	3	0.08	0.05	5
Fuzzy	30005	2.9925	0.0795	0.05025	5.025
Intuitionistic	30005.625	2.99875	0.079875	0.050375	5.0725

For stable evaluation, define $J(t) = at + bt^2/2 + ct^3/3$ and the inventory areas

$$H_R = \frac{\int_0^{t_1} e^{\alpha_R u} D(u)\, du - J(t_1)}{\alpha_R}, \qquad H_W = \frac{W_0 - J(t_2) + J(t_1)}{\alpha_W}.$$

The backlog area is $H_B = \int_{t_2}^{T} (T-u) D(u)\, du$. For each effective cost vector, the objective is evaluated as

$$\frac{A_0 + (\beta_1 + d_c\alpha_R)H_R + (\beta_2 + d_c\alpha_W)H_W + C_s H_B}{T}.$$

This expression is identical to (6). Solving the warehouse equality first avoids treating t_1 and t_2 as independent decisions. At a fixed t_1, the derivative of the numerator of the first-order condition for T is $C_s T D(T) > 0$, so the optimal $T \geq t_2$ is unique. Its equation is

$$C_s T[J(T) - J(t_2)] - A_0 - (\beta_1 + d_c\alpha_R)H_R - (\beta_2 + d_c\alpha_W)H_W - C_s H_B = 0.$$

The remaining one-dimensional search over t_1 is checked against constrained optimization of (t_1, t_2, T) from three starting points. The resulting costs agree within 2×10^{-6}. This is a numerical verification; a general uniqueness theorem for the full schedule is not assumed.

Table 2. Computed schedules for the baseline inputs.

Assessment	t_1	t_2	T^*	Average cost	Q
Crisp	13.48492	15.26625	16.36805	2589.727	6193.276
Fuzzy	13.51181	15.28614	16.38039	2588.692	6210.250
Intuitionistic	13.50000	15.27740	16.36378	2591.302	6197.255

5| Sensitivity Analysis

Each demand coefficient is varied separately while all other inputs, including the cost spreads, remain fixed at the baseline values. The warehouse equality and both time decisions are resolved at every point. The supplementary program provides the unrounded results.

5.1| Crisp Cost Assessment

a	15	18	21	24	27	30	33	36	39	42	45
T^*	16.46927	16.44894	16.42866	16.40841	16.38821	16.36805	16.34794	16.32787	16.30784	16.28785	16.26791
Average cost	2554.404	2561.491	2568.567	2575.632	2582.685	2589.727	2596.757	2603.777	2610.785	2617.781	2624.767

Table 3. Crisp cost assessment: variation of a.

b	11	11.2	11.4	11.6	11.8	12	12.2	12.4	12.6	12.8	13
T^*	16.50381	16.47632	16.44900	16.42186	16.39487	16.36805	16.34140	16.31490	16.28857	16.26239	16.23637
Average cost	2563.815	2569.037	2574.239	2579.421	2584.584	2589.727	2594.850	2599.955	2605.040	2610.106	2615.154

Table 4. Crisp cost assessment: variation of b.

c	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5
T^*	17.86800	17.50144	17.17474	16.88044	16.61296	16.36805	16.14238	15.93328	15.73863	15.55665	15.38590
Average cost	2410.641	2450.921	2488.638	2524.160	2557.779	2589.727	2620.193	2649.334	2677.284	2704.154	2730.040

Table 5. Crisp cost assessment: variation of c.

5.2| Fuzzy cost assessment

a	15	18	21	24	27	30	33	36	39	42	45
T^*	16.48152	16.46121	16.44094	16.42071	16.40053	16.38039	16.36029	16.34023	16.32022	16.30025	16.28033
Average cost	2553.448	2560.519	2567.579	2574.628	2581.665	2588.692	2595.706	2602.710	2609.703	2616.684	2623.654

Table 6. Fuzzy cost assessment: variation of a.

b	11	11.2	11.4	11.6	11.8	12	12.2	12.4	12.6	12.8	13
T^*	16.51614	16.48866	16.46134	16.43419	16.40721	16.38039	16.35373	16.32724	16.30090	16.27472	16.24870
Average cost	2562.811	2568.026	2573.222	2578.398	2583.555	2588.692	2593.809	2598.907	2603.987	2609.047	2614.089

Table 7. Fuzzy cost assessment: variation of b.

5.3| Intuitionistic Cost Assessment

c	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5
T^*	17.88189	17.51492	17.18788	16.89328	16.62554	16.38039	16.15450	15.94521	15.75037	15.56823	15.39732
Average cost	2409.622	2449.900	2487.614	2523.134	2556.749	2588.692	2619.152	2648.289	2676.233	2703.097	2728.978

Table 8. Fuzzy cost assessment: variation of c.

a	15	18	21	24	27	30	33	36	39	42	45
T^*	16.46492	16.44461	16.42434	16.40411	16.38392	16.36378	16.34368	16.32362	16.30360	16.28363	16.26370
Average cost	2555.981	2563.068	2570.143	2577.208	2584.261	2591.302	2598.332	2605.351	2612.359	2619.356	2626.341

Table 9. Intuitionistic cost assessment: variation of a.

b	11	11.2	11.4	11.6	11.8	12	12.2	12.4	12.6	12.8	13
T^*	16.49945	16.47198	16.44468	16.41755	16.39058	16.36378	16.33714	16.31066	16.28434	16.25818	16.23218
Average cost	2565.386	2570.609	2575.812	2580.995	2586.158	2591.302	2596.427	2601.532	2606.618	2611.685	2616.734

Table 10. Intuitionistic cost assessment: variation of b.

c	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5
T^*	17.86302	17.49664	17.17010	16.87593	16.60858	16.36378	16.13820	15.92920	15.73463	15.55273	15.38205
Average cost	2412.146	2452.442	2490.174	2525.710	2559.342	2591.302	2621.780	2650.933	2678.893	2705.773	2731.668

Table 11. Intuitionistic cost assessment: variation of c.

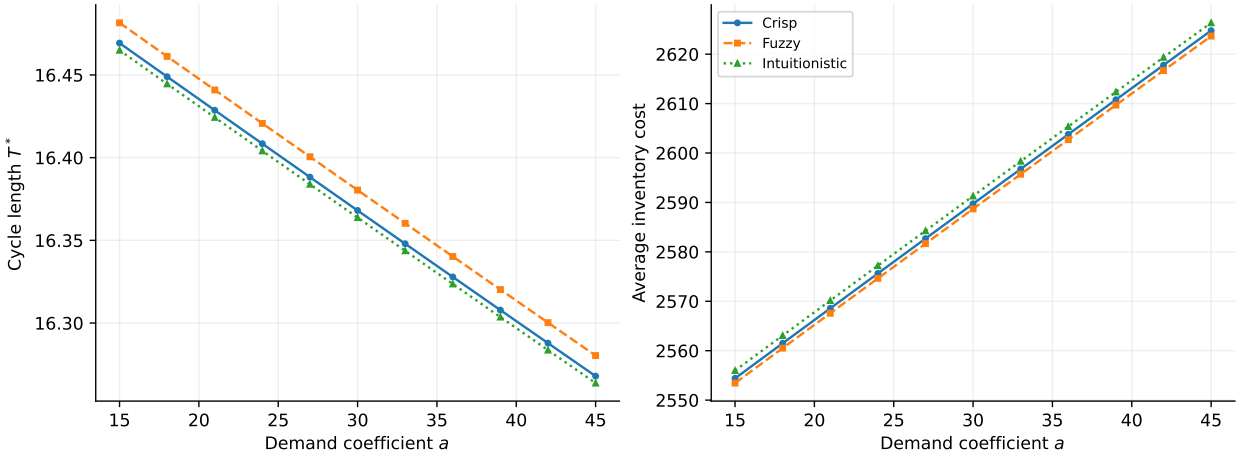


Figure 2. Computed cycle length and average cost as a varies.

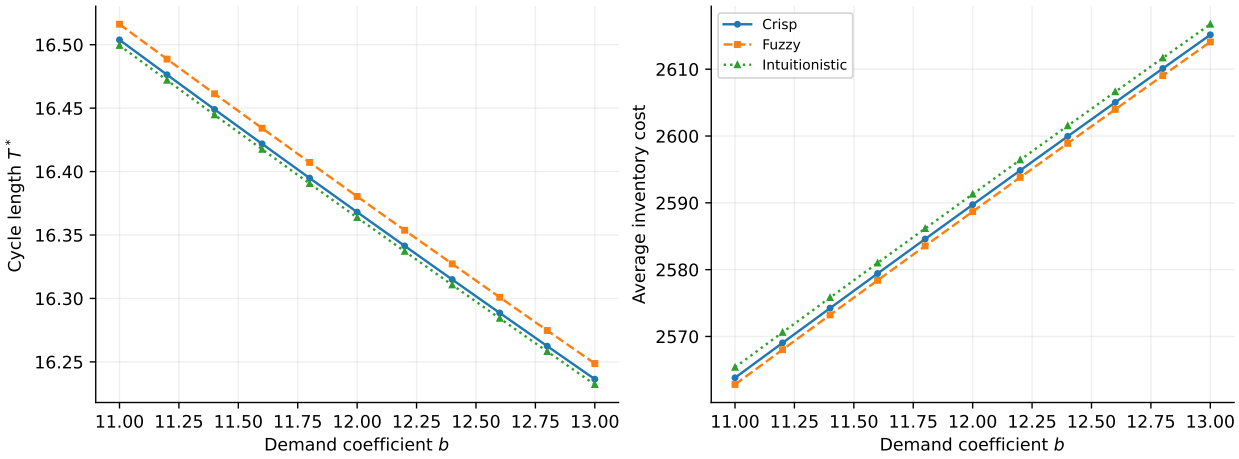


Figure 3. Computed cycle length and average cost as b varies.

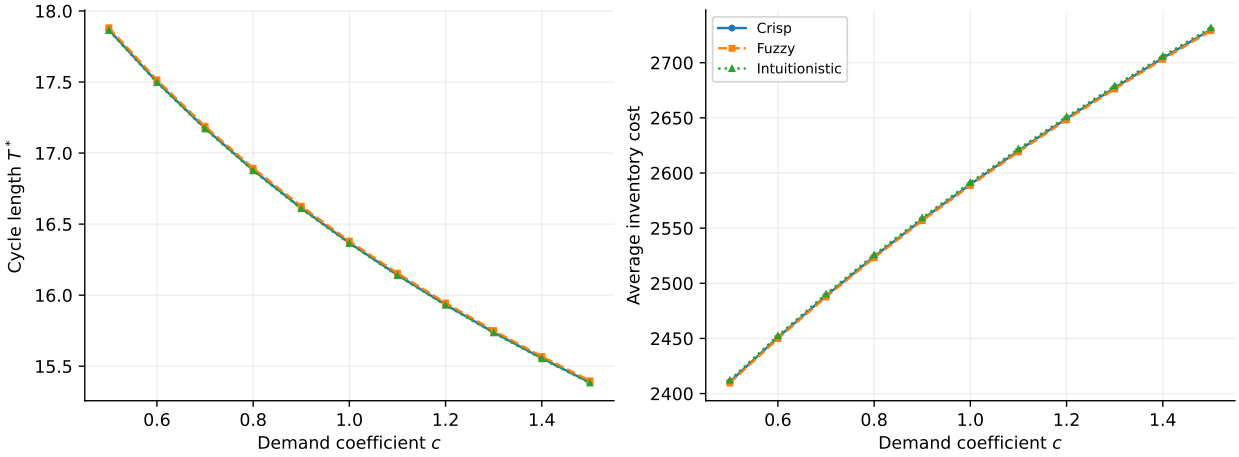


Figure 4. Computed cycle length and average cost as c varies.

5.4| Results and Discussion

Across the displayed grids, increasing a, b or c increases the computed average cost and reduces the replenishment cycle. The response to c is nonlinear. The fuzzy assessment gives the lowest cost at every displayed point, while the intuitionistic assessment gives the highest. This ordering follows from the specified effective cost vectors and is not a general ordering of crisp, fuzzy and intuitionistic models. In particular, the larger intuitionistic shortage coefficient changes both the cost and the cycle length.

For any fixed feasible schedule, the average cost is nondecreasing in each scalar cost coefficient because H_R, H_W, H_B and the ordering coefficient $1/T$ are nonnegative. Minimizing over the same feasible schedules preserves this nondecreasing property. Strict increase is not guaranteed when the relevant inventory area is zero. Differences between the three assessments therefore reflect the chosen coefficient rankings rather than a reduction in the physical amount of deterioration caused by fuzzification.

6| Conclusion

The formulation combines two warehouse balances, quadratic demand and fully backlogged shortages with three representations of uncertain costs. Enforcing the warehouse matching condition is necessary for a physically feasible schedule and nonnegative inventory areas. Signed-distance rankings lead to the same average-cost formula with different effective coefficients. The numerical example reports the complete schedule and order quantity, and the sensitivity calculations use the same coefficients consistently throughout. The computed differences depend on the stated cost spreads. Further study may consider partial backlogging, measured seasonal demand, stock-dependent demand, finite product lifetimes and time-dependent deterioration.

Data and Computation

The example inputs are stated in the article. The supplementary files include the numerical program, the complete sensitivity results and the plotting procedure. The computations use Python, NumPy, SciPy and Matplotlib.

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